THE UNIVERSITY OF NEW SOUTH WALES

SEMESTER 2 2018

COMP6741: PARAMETERIZED AND EXACT COMPUTATION

TRIAL mid-session Quiz

1. TIME ALLOWED – 90 minutes
2. READING TIME – 0 minutes
3. THIS EXAMINATION PAPER HAS 3 PAGES
4. TOTAL NUMBER OF QUESTIONS – 4
5. TOTAL MARKS AVAILABLE – 100
6. ALL QUESTIONS ARE NOT OF EQUAL VALUE. MARKS AVAILABLE FOR EACH QUESTION ARE SHOWN IN THE EXAMINATION PAPER.
7. ALL ANSWERS MUST BE WRITTEN IN INK. EXCEPT WHERE THEY ARE EXPRESSLY REQUIRED, PENCILS MAY BE USED ONLY FOR DRAWING, SKETCHING OR GRAPHICAL WORK.
8. THIS PAPER MAY NOT BE RETAINED BY CANDIDATE.

SPECIAL INSTRUCTIONS

9. ANSWER ALL QUESTIONS.
10. CANDIDATES MAY BRING TO THE EXAMINATION: UNSW approved calculator, all textbooks and lecture notes (handwritten or printed), private documents, etc., but no electronic material and no other electronic devices.
11. THE FOLLOWING MATERIALS WILL BE PROVIDED: answer booklet

Your answers may rely on theorems, lemmas and results stated in the lecture notes.

1 Basics of Parameterized Complexity [20 marks]

Prove the following theorem.

Theorem 1. *Let Π be a parameterized decision problem. If Π is* FPT*, then there exists a computable function f such that Π can be solved in time $f(k) + n^{O(1)}$, where k is the parameter and n is the instance size.*

2 Max Cut [40 marks]

A *cut* in a graph $G = (V, E)$ is a partition of the vertex set V into two sets U and W. The *size of a cut* is the number of edges with one endpoint in U and the other endpoint in W, i.e., $|\{uv \in E : u \in U \text{ and } v \in W\}|$. Consider the Max Cut problem.

Max Cut	
Input:	A graph $G = (V, E)$, an integer k
Parameter:	k
Question:	Does G have a cut of size at least k?

1. Design a simplification rule that removes vertices of degree 0. [10 marks]
2. Design a simplification rule that removes vertices of degree 1. [10 marks]
3. Obtain a kernel with $O(k)$ vertices and edges. You may use the following theorem without proving its correctness: [20 marks]

   Theorem 2. *Let $G = (V, E)$ be a graph. There is a function $\alpha : V \to \{0, 1\}$ assigning the label 0 or 1 to each vertex in V such that at least $|E|/2$ edges have one endpoint labeled 0 and the other endpoint labeled 1.* [1]

3 Kernel Lower Bound [20 marks]

Recall that a *clique* in a graph $G = (V, E)$ is a subset of vertices $S \subseteq V$ such that every two vertices from S are adjacent in G. Consider the NP-complete Generalized Edge Clique Cover problem.

Generalized Edge Clique Cover (GECC)	
Input:	A graph $G = (V, E)$, a subset of edges $R \subseteq E$, and an integer $k \leq |V|$
Parameter:	k
Question:	Is there a set $\mathcal{C}$ of at most k cliques in G such that each $e \in R$ is contained in at least one of these cliques?

- Prove that GECC has no polynomial kernel unless coNP $\subseteq$ NP/poly.

[1] **Aside:** The theorem can be proved by a simple probabilistic argument: If we randomly label the vertices of G with 0 and 1, the expected number of edges where the endpoints have distinct labels is $|E|/2$. Therefore, there exists at least one labeling where at least $|E|/2$ edges have distinct labels on their endpoints.

4 W[1]-hardness **[20 marks]**

We denote by $G = (A \uplus B, E)$ a *bipartite graph* whose vertex set is partitioned into two independent sets A and B. Consider the Hall Set problem, which asks for a subset S of at most k vertices in A whose neighborhood is smaller than S.

Hall Set (HS)	
Input:	A bipartite graph $G = (A \uplus B, E)$ and an integer k
Parameter:	k
Question:	Is there a set $S \subseteq A$ of size at most k such that $\lvert N(S)\rvert < \lvert S\rvert$?

- Show that Hall Set is W[1]-hard.

Hints: Reduce from Clique. For a set E' of edges, the set $V(E') = \{u \in e : e \in E'\}$ denotes the set of endpoints of E'. Observe that for a set E' of $\binom{k}{2}$ edges, we have that $\lvert V(E')\rvert \leq k$ if and only if $V(E')$ is a clique of size k.

End of Paper