Exercise sheet 5 – Solutions and Hints

COMP6741: Parameterized and Exact Computation

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Exercise 1. A *dominating set* of a graph $G = (V, E)$ is a set of vertices $S \subseteq V$ such that $N_G[S] = V$.

DEGREE-5 DOMINATING SET	
Input:	A graph $G = (V, E)$ with maximum degree at most 5 and an integer k
Parameter:	k
Question:	Does G have a dominating set of size at most k?

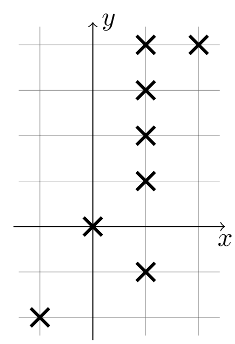
Design a linear kernel for DEGREE-5 DOMINATING SET.

Solution sketch. Simplification rule: If $|V| > 6 \cdot k$, then return NO.

Exercise 2. Consider the following problem.

POINT LINE COVER	
Input:	A set of points P in $\mathbb{Z}^2$, and an integer k
Parameter:	k
Question:	Is there a set L of at most k lines in $\mathbb{R}^2$ such that each point in P lies on at least one line in L?

Example: $(P = \{(-1,-2),(0,0),(1,-1),(1,1),(1,2),(1,3),(1,4),(2,4)\}, k = 2)$ is a Yes-instance since the lines $y = 1$ and $y = 2x$ cover all the points.



Show that POINT LINE COVER has a polynomial kernel.

Hint.
(1) Show that the algorithm can restrict its attention to a polynomial number of *candidate lines* (aim for $O(|P|^2)$).
(2) Design a simplification rule for the case where one candidate line covers many points in P.
(3) Design a simplification rule that solves POINT LINE COVER when $|P|$ is large compared to t.

Exercise 3. A *cluster graph* is a graph where every connected component is a complete graph.

CLUSTER EDITING	
Input:	Graph $G = (V, E)$, integer k
Parameter:	k
Question:	Is it possible to edit (add or delete) at most k edges of G so that it becomes a cluster graph?

1. Show that G is a cluster graph iff G contains no induced P_3 (path with 3 vertices).
2. Design a kernel for Cluster Editing with $O(k^2)$ vertices.

Hint. Design simplification rules for (1) a vertex that does not occur in any P_3, (2) an edge that occurs in many P_3s, and (3) a non-edge that occurs in many P_3s

Exercise 4. A *k-coloring* of a graph $G = (V, E)$ is a function $f : V \to \{1, 2, ..., k\}$ such that $f(u) \neq f(v)$ if $uv \in E$.

Saving Colors	
Input:	Graph G, integer k
Parameter:	k
Question:	Does G have a $(n-k)$-coloring?

Design a kernel for Saving Colors with $O(k)$ vertices.Recommendation: use the Crown Lemma.

Hint. Get rid of vertices v with $N_G[v] = V$ and consider the dual of G, i.e., the graph $\overline{G} = (V, \{uv : u, v \in V \text{ and } uv \notin E\})$. Use the Crown Lemma with $\overline{G}$ and $k-1$.

Exercise 5. An *edge clique cover* of a graph G is a set of cliques in G so that each edge of G is contained in at least one of these cliques.

Edge Clique Cover	
Input:	graph G, integer k
Parameter:	k
Question:	Does G have an edge clique cover with k cliques?

Design a kernel for Edge Clique Cover with $O(2^k)$ vertices.

Hint. Consider 2 vertices that are contained in exactly the same cliques.