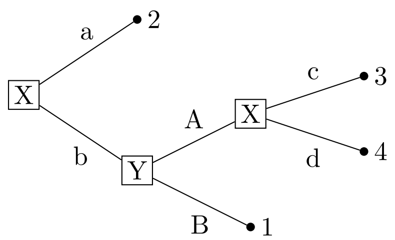
GSOE9210 Engineering Decisions

Problem Set 09

1. Consider the following game tree for a zero-sum game (payoffs shown are for player X):



   (a) What are the possible strategies for players X and Y?
   (b) Draw the corresponding normal (matrix) form from the perspective of player X.
   (c) Rationalise the game by eliminating strategies that aren't best responses.

2. Find all saddle points for the zero-sum game with the following matrix:

	b_1	b_2	b_3	b_4
a_1	6	-1	1	0
a_2	5	1	7	-2
a_3	3	2	4	3
a_4	-1	0	0	6

3. Find all saddle points for the following zero-sum game:

	b_1	b_2	b_3	b_4
a_1	4	2	5	2
a_2	2	1	-1	-2
a_3	3	2	4	2
a_4	-1	0	6	1

4. Consider the game matrix below for a non zero-sum game in which the row player is R and the column player is C.

	b_1	b_2	b_3
a_1	$0,0$	$1,2$	$0,2$
a_2	$1,3$	$1,4$	$0,0$

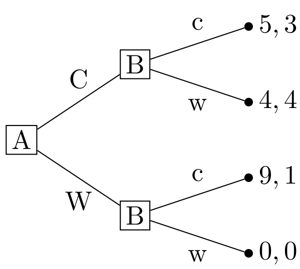
(a) Identify the equilibrium plays.
(b) Which plays are rationalisable?
(c) Which plays are Pareto optimal?

5. Show that in a zero-sum game every outcome is Pareto optimal.

6. The game 'Matching Pennies' is a two-player zero-sum game in which each player simultaneous places a coin on a table with either heads or tails facing up. The 'matching' player (M) wins if the faces on the coin match, and the 'opposites' player (O) wins if the faces are opposite. The winner takes both coins.

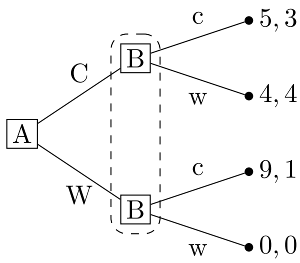
   (a) Represent this game in normal form, showing M's payoffs.
   (b) Reduce the game using dominance.
   (c) Find the equilibrium points of this game.
   (d) If M believes that O is twice as likely to play heads than tails, what would be his best response?
   (e) What should be M's best response if he believes O will play heads and tails with equal likelihood?
   (f) Repeat the above for O.
   (g) If the game is repeated many times, what strategy should the players play?

7. The following game tree represent the example in lectures for the case in which Alice the gorilla moves first and Bob the monkey moves second.



   (a) What is Bob's best response to Alice if she waits? If Alice climbs?
   (b) Are there any equilibrium points? If so, which are they?
   (c) Rationalise the game using dominance.

8. If Alice and Bob move simultaneously we have the game tree below.



(a) Draw the game matrix.

(b) Are there any equilibrium plays/points? If so, which are they?

(c) If Bob had an injured arm, ‘signalling’ that he would be less likely to climb than to wait, which would be the rational solution?

9. Show that if a zero-sum game has a dominant row and column, then the two determine an equilibrium play.