Exercise 1. A dominating set of a graph $G = (V, E)$ is a set of vertices $S \subseteq V$ such that $N_G[S] = V$.

**Degree-5 Dominating Set**

**Input:** A graph $G = (V, E)$ with maximum degree at most 5 and an integer $k$

**Parameter:** $k$

**Question:** Does $G$ have a dominating set of size at most $k$?

Design a linear kernel for Degree-5 Dominating Set.

Exercise 2. Consider the following problem.

**Point Line Cover**

**Input:** A set of points $P$ in $\mathbb{Z}^2$, and an integer $k$

**Parameter:** $k$

**Question:** Is there a set $L$ of at most $k$ lines in $\mathbb{R}^2$ such that each point in $P$ lies on at least one line in $L$?

Example: $(P = \{(-1,-2), (0,0), (1,-1), (1,1), (1,2), (1,3), (1,4), (2,4)\}, k = 2)$ is a Yes-instance since the lines $y = 1$ and $y = 2x$ cover all the points.

Show that Point Line Cover has a polynomial kernel.

Exercise 3. A cluster graph is a graph where every connected component is a complete graph.

**Cluster Editing**

**Input:** Graph $G = (V, E)$, integer $k$

**Parameter:** $k$

**Question:** Is it possible to edit (add or delete) at most $k$ edges of $G$ so that it becomes a cluster graph?

1. Show that $G$ is a cluster graph iff $G$ contains no induced $P_3$ (path with 3 vertices).

2. Design a kernel for Cluster Editing with $O(k^2)$ vertices.
Exercise 4. A k-coloring of a graph $G = (V, E)$ is a function $f : V \rightarrow \{1, 2, ..., k\}$ such that $f(u) \neq f(v)$ if $uv \in E$.

<table>
<thead>
<tr>
<th>SAVING COLORS</th>
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<tbody>
<tr>
<td>Input: Graph $G$, integer $k$</td>
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<td>Parameter: $k$</td>
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<tr>
<td>Question: Does $G$ have a $(n - k)$-coloring?</td>
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Design a kernel for SAVING COLORS with $O(k)$ vertices. Recommendation: use the Crown Lemma.

Exercise 5. An edge clique cover of a graph $G$ is a set of cliques in $G$ so that each edge of $G$ is contained in at least one of these cliques.

<table>
<thead>
<tr>
<th>EDGE CLIQUE COVER</th>
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<tbody>
<tr>
<td>Input: graph $G$, integer $k$</td>
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<td>Parameter: $k$</td>
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<td>Question: Does $G$ have an edge clique cover with $k$ cliques?</td>
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Design a kernel for EDGE CLIQUE COVER with $O(2^k)$ vertices.