Exercise sheet 5

COMP6741: Parameterized and Exact Computation

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Exercise 1. A *dominating set* of a graph $G = (V, E)$ is a set of vertices $S \subseteq V$ such that $N_G[S] = V$.

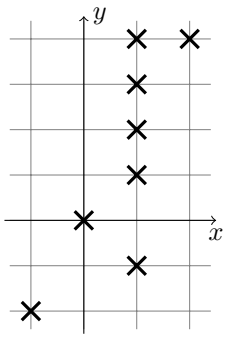
Degree-5 Dominating Set	
Input:	A graph $G = (V, E)$ with maximum degree at most 5 and an integer k
Parameter:	k
Question:	Does G have a dominating set of size at most k?

Design a linear kernel for Degree-5 Dominating Set.

Exercise 2. Consider the following problem.

Point Line Cover	
Input:	A set of points P in $\mathbb{Z}^2$, and an integer k
Parameter:	k
Question:	Is there a set L of at most k lines in $\mathbb{R}^2$ such that each point in P lies on at least one line in L?

Example: $(P = \{(-1,-2),(0,0),(1,-1),(1,1),(1,2),(1,3),(1,4),(2,4)\}, k = 2)$ is a Yes-instance since the lines $y = 1$ and $y = 2x$ cover all the points.



Show that Point Line Cover has a polynomial kernel.

Exercise 3. A *cluster graph* is a graph where every connected component is a complete graph.

Cluster Editing	
Input:	Graph $G = (V, E)$, integer k
Parameter:	k
Question:	Is it possible to edit (add or delete) at most k edges of G so that it becomes a cluster graph?

1. Show that G is a cluster graph iff G contains no induced P_3 (path with 3 vertices).
2. Design a kernel for Cluster Editing with $O(k^2)$ vertices.

Exercise 4. A *k-coloring* of a graph $G = (V, E)$ is a function $f : V \to \{1, 2, ..., k\}$ such that $f(u) \neq f(v)$ if $uv \in E$.

Saving Colors	
Input:	Graph G, integer k
Parameter:	k
Question:	Does G have a $(n - k)$-coloring?

Design a kernel for Saving Colors with $O(k)$ vertices.Recommendation: use the Crown Lemma.

Exercise 5. An *edge clique cover* of a graph G is a set of cliques in G so that each edge of G is contained in at least one of these cliques.

Edge Clique Cover	
Input:	graph G, integer k
Parameter:	k
Question:	Does G have an edge clique cover with k cliques?

Design a kernel for Edge Clique Cover with $O(2^k)$ vertices.