Exercise sheet 8

COMP6741: Parameterized and Exact Computation

Serge Gaspers

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Exercise 1. Recall that a *k-coloring* of a graph $G = (V, E)$ is a function $f : V \to \{1, 2, ..., k\}$ assigning colors to V such that no two adjacent vertices receive the same color.

COLORING	
Input:	Graph G, integer k
Question:	Does G have a k-coloring?

- Suppose A is an algorithm solving COLORING in $O(f(n))$ time, $n = |V|$, where f is non-decreasing. Design a $O^*(f(n))$ time algorithm B, which, for an input graph G, finds a coloring of G with a smallest number of colors.

Exercise 2. Recall that a graph $G = (V, E)$ is *bipartite* if G has a 2-coloring. A *matching* in a graph $G = (V, E)$ is a set of edges $M \subseteq E$ such that no two edges of M have an end-point in common. The matching M in G is *perfect* if every vertex of G is contained in an edge of M.

#BIPARTITE PERFECT MATCHINGS	
Input:	Bipartite graph $G = (V, E)$
Output:	The number of perfect matchings in G

1. Design an algorithm for #BIPARTITE PERFECT MATCHINGS with running time $O^*\left(\left(\frac{n}{2}\right)!\right)$, where $n = |V|$.
2. Design a polynomial-space $O^*(2^{n/2})$-time inclusion-exclusion algorithm for #BIPARTITE PERFECT MATCHINGS.