COMP9020 Lecture 12
Session 2, 2017
Course Review

Course Review

Goal: for you to become a competent computer **scientist**.

Requires an understanding of fundamental concepts:

- number-, set-, relation- and graph theory
- logic and proofs
- recursion and induction
- order of growth of functions
- combinatorics and probability

In CS/CE these are used to:

- formalise problem specifications and requirements
- develop abstract solutions (algorithms)
- analyse and prove properties of your programs

Assessment Summary

Quizzes: Best 5 out of 10 – (10 marks)
Assignments: 3 assignments worth 10 marks each – (30 marks)
Final exam – (60 marks)

NB

You must achieve 40% on the final exam AND 50% overall to pass the course.

Final Exam

Goal: to check whether you are a competent computer scientist.

Requires you to demonstrate:

- understanding of mathematical concepts
- ability to apply these concepts and explain how they work

Lectures, quizzes and assignments have built you up to this point.

Final Exam

Friday, 3 November, 8:45am
Multiple locations

- 10 multiple-choice questions plus 5 open questions
- Covers **all** of the contents of this course up to expectation.
- Each multiple-choice question is worth 3 marks ($10 \times 3 = 30$)
  Each open question is worth 18 marks ($5 \times 18 = 90$)
  Total exam marks = 120 (i.e. 1 mark/minute)
- Time allowed – 120 minutes + 10 minutes reading time
- *Closed book*. One handwritten or typed A4-sized sheet (double-sided is ok) of your own notes
- Two answer booklets – one for rough work and one for additional solution space (answers should begin on exam paper). Clearly indicate which one is which.

Revision Strategy

- Re-read lecture slides
- Read the corresponding chapters in the book (R & W)
- **Review/solve assignments and quizzes**
- Review/solve problem sets
- Solve more problems from the book
- Attempt practice exam on course webpage
- Practice multiple choice questions

(Applying mathematical concepts to solve problems is a skill that improves with practice)

Additional consultations:

-
- Thursday, 2 November, 12 noon – 4:00pm K17, Meeting Room 402

Supplementary Exam

You can apply formally for special consideration

- a supplementary examination may or may not be granted
- a supplementary examination is typically more difficult than the original examination

If you attend an exam

- you are making a statement that you are "fit and healthy enough"
- it is your only chance to pass (i.e. no second chances)

If your overall result is $\geq$ **47** you can sit the supplementary exam, in which you must score 50 or higher to pass

Assessment

Assessment is about determining how well you understand the syllabus of this course.

If you can't demonstrate your understanding, you don't pass.

In particular, I can't pass people just because ...

- please, please, ... my family/friends will be ashamed of me
- please, please, ... I tried really hard in this course
- please, please, ... I'll be excluded if I fail COMP9020
- please, please, ... this is my final course to graduate
- etc. etc. etc.

(Failure is a fact of life. For example, my scientific papers or project proposals get rejected sometimes too)

Assessment (cont'd)

Of course, assessment isn't a "one-way street" ...

- I get to assess you in the final exam
- you get to assess me in UNSW's MyExperience Evaluation
  - go to `https://myexperience.unsw.edu.au/`
  - login using zID@ad.unsw.edu.au and your zPass

Please fill it out ...

- give me some feedback on how you might like the course to run in the future
- even if that is "Exactly the same. It was perfect this time."

Assessment insight

Example (Q1 Assignment 2)

Eight houses are lined up on a street, with four on each side of the road as shown:

Each house wants to set up its own wi-fi network, but the wireless networks of neighbouring houses – that is, houses that are either next to each other (ignoring trees) or over the road from one another (directly opposite) – can interfere, and must therefore be on different channels. Houses that are sufficiently far away may use the same wi-fi channel. Your goal is to find the minimum number of different channels the neighbourhood requires.

Assessment insight

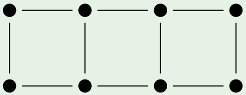
Example (Q1 Assignment 2)

Explain how this can be formulated as a graph-based problem.

The problem can be viewed as the problem of finding the chromatic number (i.e. minimum number of colours required to colour vertices such that no edge has two end-points the same colour) of the following graph:

- We have one vertex for every house
- We have an edge between houses if, and only if, their wireless networks can interfere.

In this example the graph would be:

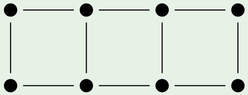


Assessment insight

Example (Q1 Assignment 2)

Explain how this can be formulated as a graph-based problem.

In this example the graph would be:



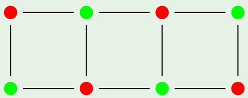
Each colour in a valid colouring represents the wi-fi channel that the house will use - so a valid colouring corresponds to a non-interfering assignment of wi-fi channels. Finding the minimum number of different channels is therefore the same as finding the minimum number of colours to validly colour the graph.

Assessment insight

Example (Q1 Assignment 2)

What is the minimum number of wi-fi channels required for the neighbourhood?

Here is a 2-colouring of the graph from part (a):



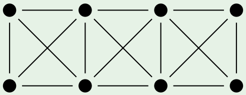
Since the graph has an edge, we cannot colour the graph with fewer than 2 colours, so the minimum number of wi-fi channels required for the neighbourhood is 2.

Assessment insight

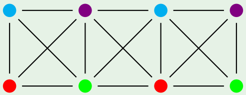
Example (Q1 Assignment 2)

How does your answer to (b) change if a house's wireless network can also interfere with those of the houses to the left and right of the house over the road?

With the additional constraint, the neighbourhood interference graph becomes:



Here is a 4-colouring of this graph:



The graph contains K_4, so we cannot colour it with fewer than 4 colours, so 4 is the minimum number of wi-fi channels.

Assessment insight

Example (Q4 Assignment 2)

We can define addition over natural numbers inductively as follows:

- $\text{add}(m, 0) = m$, and
- $\text{add}(m, n + 1) = \text{add}(m, n) + 1$.

We will now show that add is commutative!

1. Let $P(n)$ be the proposition

$$P(n): \ \text{add}(n, 0) = \text{add}(0, n).$$

   Prove that $P(n)$ holds for all $n \in \mathbb{N}$.

2. Let $Q(n)$ be the proposition

$$Q(n): \ \text{If } a + b = n \text{ and } a, b > 0 \text{ then } \text{add}(a, b) = \text{add}(b, a).$$

   Prove that $Q(n)$ holds for all $n \in \mathbb{N}$.

Assignment 2, Q4

We will prove that $P(n)$ holds for all $n \in \mathbb{N}$ by induction on n.
Base case. $\text{add}(0,0) = \text{add}(0,0)$ so $P(0)$ holds.
Inductive case. Suppose $P(k)$ holds for $k \geq 0$. That is $\text{add}(k,0) = \text{add}(0,k)$. Then

$$
\begin{aligned}
\text{add}(0, k+1) &= \text{add}(0,k) + 1 && \text{Definition of add} \\
&= \text{add}(k,0) + 1 && \text{IH} \\
&= k + 1 && \text{Definition of add} \\
&= \text{add}(k+1, 0). && \text{Definition of add}
\end{aligned}
$$

So $P(k+1)$ holds.
We have shown $P(0)$ and $P(k) \rightarrow P(k+1)$ for $k \geq 0$, so $P(n)$ holds for all $n \in \mathbb{N}$.

Assignment 2, Q4

We will prove that $Q(n)$ holds for all $n \in \mathbb{N}$ by strong induction on n.

Base case. When $n = 0$ there is no a or b with $a, b > 0$ such that $a + b = n$. So $Q(0)$ is true vacuously.

Inductive case. Suppose $Q(k')$ holds for all $k' < k$. From (a) we also have that $P(n)$ holds for all $n \in \mathbb{N}$. Let $a + b = k$ with $a, b > 0$. Let $a = a' + 1$ and $b = b' + 1$ where $a', b' \geq 0$. From the inductive hypothesis and $P(n)$, regardless of a and b, we have:

$$\text{add}(a', b) = \text{add}(b, a');\ \text{add}(a, b') = \text{add}(b', a);\ \text{add}(a', b') = \text{add}(b', a').$$

Assignment 2, Q4

$\text{add}(a', b) = \text{add}(b, a');\ \text{add}(a, b') = \text{add}(b', a);\ \text{add}(a', b') = \text{add}(b', a')$.

$$
\begin{aligned}
\text{add}(a, b) &= \text{add}(a, b' + 1) && \text{Definition of } b' \\
&= \text{add}(a, b') + 1 && \text{Definition of add} \\
&= \text{add}(b', a) + 1 && \text{From above} \\
&= \text{add}(b', a' + 1) + 1 && \text{Definition of } a' \\
&= \text{add}(b', a') + 1 + 1 && \text{Definition of add} \\
&= \text{add}(a', b') + 1 + 1 && \text{From above} \\
&= \text{add}(a', b' + 1) + 1 && \text{Definition of add} \\
&= \text{add}(a', b) + 1 && \text{Definition of } b' \\
&= \text{add}(b, a') + 1 && \text{From above} \\
&= \text{add}(b, a' + 1) && \text{Definition of add} \\
&= \text{add}(b, a). && \text{Definition of } a'
\end{aligned}
$$

We note that this holds for any $a, b > 0$ with $a + b = k$, therefore $Q(k)$ holds.

Assignment 2, Q4

Alternative approach:

- Prove by induction on n that $\text{add}(a, n) = a + n$
- $\text{add}(a, b) = a + b = b + a = \text{add}(b, a)$

Subtle issue (minor penalty in assignment, would not be penalized in exam):

- $a + b = (((a + 1) + 1) + \ldots) + 1$
- $b + a = (((b + 1) + 1) + \ldots) + 1$

Content review

Numbers, Sets, and Words

- Divides, gcd, lcm
- Union, intersection, subset
- Cardinality, Power set, Cartesian product
- Venn diagrams, Set operation laws (see Boolean algebras)
- λ, Σ^*, $\Sigma^{\leq k}$, Σ^k
- lexicographic, lenlex ordering

Functions and relations

Relation, R, between A and B: $R \subseteq A \times B$

-
- *Converse*, $R^{\leftarrow} \subseteq B \times A$, $R^{\leftarrow} = \{(b, a) : (a, b) \in R\}$.
- Image: $R(X) \subseteq B$

Function $f : A \to B$: Relation that relates each element of the domain (A) to exactly one element of the co-domain (B).

- Image (Range)
- Injection, surjection, bijection
- Inverse $f^{-1} : B \to A$ (exists if and only if f is a bijection)
- Inverse image $f^{\leftarrow}(X)$ – image of X under converse of f

Binary relations

Binary relation on A, $R \subseteq A \times A$

- Reflexivity, Symmetry, Antisymmetry, Transitivity
- Equivalence relations and equivalence classes
- Partial orders
  - glb, lub
  - Hasse diagrams
  - Topological sort

NB

Reflexivity, Symmetry, Antisymmetry, Transitivity must be shown to hold **for all** *elements.*
Counter-examples only require one case.
(E.g. Quiz 8, Q2)

Graphs

- Basic definitions
- Eulerian circuit/path, Hamiltonian circuit/path
- Graph colouring, chromatic number
- Planarity

Logic

- Conjunction, Disjunction, Negation, etc (Well-formed formulas)
- Logical equivalence $\equiv$, Logical entailment $\models$
- Satisfaction, validity
- CNF/DNF
- Boolean algebras

Example Question

Example

Show that logical equivalence is an equivalence relation.

Induction and Recursion

- Do not need to worry about all the different types of induction.
- Any induction question will have one or two base cases; one or two inductive steps
- Structural induction is possible (will be simple)

Running time of algorithms

- Big-O notation
- Master theorem – when and how it applies

Counting

- k objects from n with replacement: k^n
- k objects from n without replacement: $\binom{n}{k}$
- k balls into n boxes, many balls per box $\binom{n+k-1}{k}$

Basic probability and expectation

- Sample space, uniform distribution
- Expected value, linearity of expectation

Conditional probability and independence

- Not on the exam
- $P(A|B) = P(A \cap B)/P(B)$
- A and B are independent ($A \perp B$) if $P(A \cap B) = P(A).P(B)$

Assignment review

Assignment 1: Average $\sim$85%
Assignment 2: Average $\sim$75%
Quizzes: Average $\sim 4/5$

Assignment 1

Q1: Several approaches. Intermediate steps often omitted

Q2: Two main approaches. Venn diagrams fared better than logical reasoning.

Q3: A few people forgot λ

Q4: Connection between $\mathrm{Pow}(\{a, b, c\})$ and $f : \{a, b, c\} \to \{0, 1\}$

Q5: Many people didn't apply the relation *for all v*

Assignment 2

Q1: Lower bounds for chromatic number omitted

Q2: Explanation for the formulas often omitted. CNF vs DNF issues.

Q3: Often people did not work in general Boolean Algebra setting.

Q4: Straightforward + Tricky induction. Many people did not start at $n = 0$. Subtle issue that most people did not see.

Q5: Part (b) is a good induction practice question.

All the best!