1. A carrier is planning to build a network between 4 cities, which are labelled as Cities 1, 2, 3 and 4. The amount of traffic flowing between these cities are given in Table 1. To build this network, the carrier can choose between three different types of links with different cost-capacity combinations, as showed in Table 2. Note that:

- Each link is directed, i.e. it only carries traffic in one direction.
- The decision to build links \((i, j)\) and \((j, i)\) are independent. In other words, if the carrier chooses to build link \((i, j)\), it does not have to build link \((j, i)\).
- Each link, if built, must have sufficient capacity to carry the traffic assigned to it.
- Each flow, i.e. the traffic between a pair of cities, can only use one path.

The carrier aims to build a network with the minimum cost. Formulate an integer programming problem and solve it using AMPL/CPLEX to determine:

(a) Which links the carrier should build and what type should it be?
(b) How the traffic is to be routed in the network?

Table 1: Amount of traffic between cities.

<table>
<thead>
<tr>
<th>From</th>
<th>To</th>
<th>City 1</th>
<th>City 2</th>
<th>City 3</th>
<th>City 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>City 1</td>
<td>–</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td></td>
</tr>
<tr>
<td>City 2</td>
<td>3</td>
<td>–</td>
<td>2</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>City 3</td>
<td>2</td>
<td>4</td>
<td>–</td>
<td>2</td>
<td></td>
</tr>
<tr>
<td>City 4</td>
<td>1</td>
<td>5</td>
<td>3</td>
<td>–</td>
<td></td>
</tr>
</tbody>
</table>

Table 2: Capacity and cost for different types of links.

<table>
<thead>
<tr>
<th>Type of link</th>
<th>Capacity</th>
<th>Cost</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>10</td>
<td>10</td>
</tr>
<tr>
<td>2</td>
<td>20</td>
<td>18</td>
</tr>
<tr>
<td>3</td>
<td>30</td>
<td>25</td>
</tr>
</tbody>
</table>
Hint

Define decision variables

\[ x_{ijuv} = \begin{cases} 
1 & \text{if the traffic from Node } u \text{ to Node } v \text{ uses link } (i, j) \\
0 & \text{otherwise}
\end{cases} \]

\[ y_{ijk} = \begin{cases} 
1 & \text{if a link of type } k \text{ is to be built from node } i \text{ to node } j \\
0 & \text{otherwise}
\end{cases} \]

If all \( y_{ij1}, y_{ij2} \) and \( y_{ij3} \) are zero, then this means a link won’t be built from node \( i \) to node \( j \). Impose constraints that if there isn’t a link, no traffic can use that link etc.

2. In this question, you will formulate an integer programming problem for the controller placement problem in software defined networking (SDN). We briefly discussed this problem in the beginning of the lecture in Week 10, see pages 2-3 of the lecture notes for that week.

In SDN, there are two types of “boxes”: (1) Simple packet switches; and (2) Controllers. At each node of the network, there is a simple packet switch. However, we do not need to have a controller at each network node. We only need to place controllers at some of the network nodes, and one controller can be used to control multiple switches. Since a controller needs to communicate with those switches that it controls, the communication delay between a controller and a switch under its control must not be too big.

For the controller placement problem, you are given:

- A network \((N, E)\) where \( N \) is the set of nodes and \( E \) is the set of edges. There is a simple packet switch at each node in \( N \). We assume there are \( n \) nodes and they are indexed by \( 1, 2, ..., n \).
- The communication delay \( d_{ij} \) between nodes \( i \) and \( j \) in the network, for all \( i, j \in N \). This means that if a controller is placed at node \( i \), then the communication delay between this controller and a switch at node \( j \) is \( d_{ij} \).
- The maximum allowable delay \( D \) between a controller and the switches that it controls.

The requirements are:

- Each switch must be controlled by a controller.
- The delay between a controller and any switch that it controls cannot exceed \( D \).

Formulate an integer programming to minimise the number of controllers required subject to the above requirements.