GSOE9210 Engineering Decisions

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Introduction

1. Introduction
    - Motivation
    - Decision problems: examples
    - Course overview
2. Decision problems: representation
    - Decision problem elements
    - Decision trees
    - Decision tables
        - Trees

Outline

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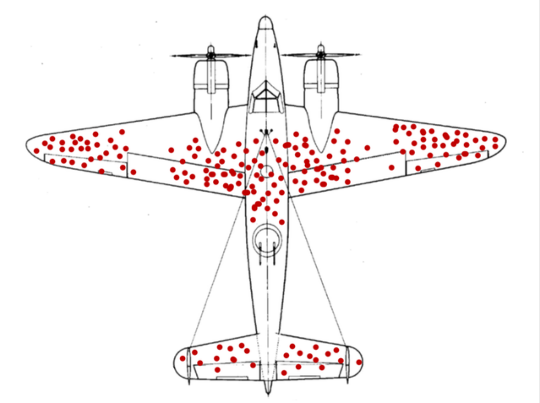
The story of the missing bullet holes

Abraham Wald (1902–1950)



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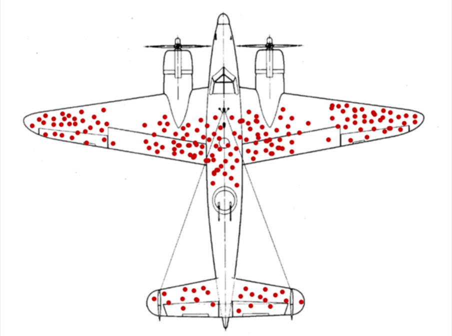
The story of the missing bullet holes …



Airforce General:

> *Professor Wald, how much armour should we add to reinforce our aircraft?*

Wald's reply:

Put the armour where there are no bullet holes.

`http://paristampablog.com/2014/09/25/abraham-wald-and-the-missing-bullet-holes/`

Decision problems

Example (Oil exploration)

You're the chief petroleum engineer of an oil company which owns a drilling option on an area of sea. You're responsible for deciding whether or not to drill before the option expires.

Considerations:

- likelihood of finding oil, amount and quality, projected oil demand
- size and location of drilling
- cost of drilling and raising the oil, *etc.*

Decision problems

Example (Drug development)

You're a chemical engineer in a major pharmaceuticals company which is considering whether to synthesise a new drug for cancer treatment. Initial findings are inconclusive as to the drug's effectiveness.

Considerations:

- likelihood of drug's effectiveness
- level of investment, timing, competition, *etc.*
- cost to the company of synthesis and trials value of human life, *etc.*

Decision problems

Example (Manufacturing processes)

You're the head process engineer of Acme Inc., a company which manufactures mechanical components for the automobile industry. Due to a new technology, there is a potential for increased demand for Acme's components in the near future. The managing director has requested a report on existing plant capacity and possible production options.

Considerations:

- likelihood and degree of increased demand
- options for increasing plant production
- cost-benefit analysis of capital investment, *etc.*

Decision problems

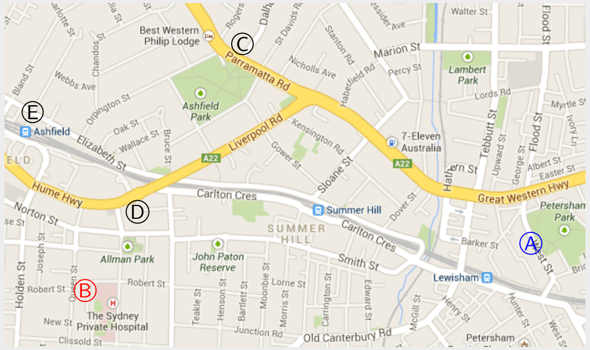
Example (To insure or not)

You own a necklace which you intend to sell at the end of the year. Should you insure it against theft?

Considerations:

- value of necklace
- cost of insurance
- likelihood of theft

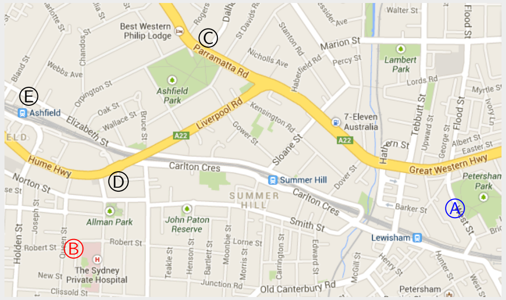
Decision problems



Example (Getting from A to B)

You have to get from Petersham Park (A) to the Hospital (B) by either train or bus. The train goes to Ashfield Station (E). You don't know the bus route: either via Parramatta Rd (C) or Liverpool Rd (D).

Decision problems: discussion



Suppose you:

- are an ER surgeon
- are a tourist
- *have an injured foot . . .*

Quantitative problems

Example (Inventory)

Your football club needs to provide playing uniforms for each of its members. The initial order needs to be placed before registrations are complete; *i.e.*, before the final number of members has been determined. The initial (early) order incurs a fee of $\$500$ plus $\$20$ per uniform. Late orders incur an additional $\$300$ fee plus the usual $\$20$ per uniform. Uniforms are sold for $\$40$ each.

How many uniforms should you order initially?

Group decisions

Example (Song contest)

Seven judges vote for four songs: A, B, C, D.

	J1	J2	J3	J4	J5	J6	J7	Tot.
A	4	1	2	4	1	2	4	18
B	3	4	1	3	4	1	3	19
C	2	3	4	2	3	4	2	20
D	1	2	3	1	2	3	1	13

What if song D is disqualified?

	J1	J2	J3	J4	J5	J6	J7	Tot.
A	3	1	2	3	1	2	3	15
B	2	3	1	2	3	1	2	14
C	1	2	3	1	2	3	1	13

Group decisions

Example

Three people (P1, P2, P3) vote for three candidates A, B, C in a poll. The preferences are:

	P1	P2	P3
1st	B	C	A
2nd	C	A	B
3rd	A	B	C

What should be the group preference?

- Most preferred, second preference, . . .
- Majority: two voters prefer B to C, two C to A, . . .

Motivation

Ron Howard, Professor of Economic-Engineering Systems at Stanford:

> *"If . . . decision-theoretic structures do not in the future occupy a large part of the education of engineers, then the engineering profession will find that its traditional role of managing scientific and economic resources for the benefit of man has been forfeited to another profession."*
>
> *—Ron Howard (1966).*

Course overview

Course aims:

> *To equip engineering graduates with analytical decision-making skills and techniques.*

Course structure:

- Single-agent decisions
- Multi-agent decisions: games

Teaching methodology:

- Mix of theoretical and applied
- Universal principles rather than domain specific knowledge

Single-agent decisions: overview

1. Decision problems
2. Decision problem representations: trees and tables
3. Decisions under uncertainty (ignorance and risk)
4. Measuring uncertainty: probability
5. Preference and utility
6. Information and its value

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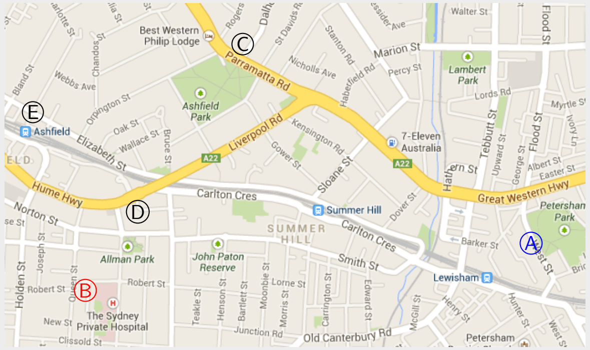
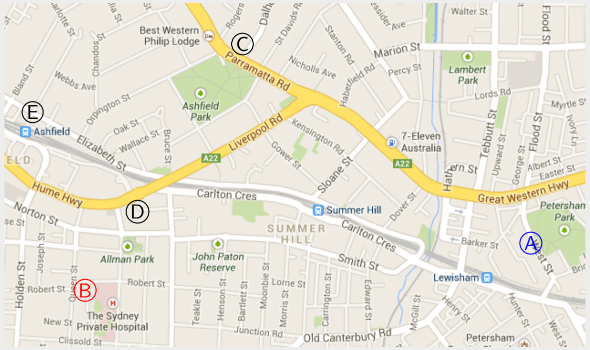
Decision problems



Example (Getting from A to B)

You have to get from Petersham Park (A) to the Hospital (B) by either train or bus. The train goes to Ashfield Station (E). You don't know the bus route: either via Parramatta Rd (C) or Liverpool Rd (D).

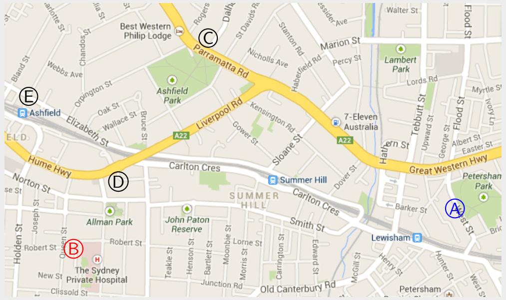
Decision problems: elements



The basic elements of a *decision problems* are:

- *actions* (acts, alternatives) ($\mathcal{A}$): Tr, Bu
- *states* (events, cases, situations, circumstances, contexts) ($\mathcal{S}$): *e.g.*, Liverpool Rd bus (b_L) or Parramatta Rd bus (b_P)
- *outcomes* (consequences) (Ω): arrive at C, D, or E

Decision problems: elements



- $\mathcal{A} = \{$Tr, Bu$\}$, $\Omega = \{$C, D, E$\}$, $\mathcal{S} = \{b_L, b_P\}$
- each action is associated with a set of possible outcomes: *e.g.*, Tr: {E}, Bu: {C, D}

Decision problems

Example (To insure or not)

You own a necklace which you intend to sell at the end of the year. Should you insure it against theft?

- *actions* ($\mathcal{A}$): Insure, don't insure
- *events* ($\mathcal{S}$): necklace stolen, necklace not stolen
- *outcomes* (Ω): uninsured necklace sold (not stolen), insured necklace sold, necklace stolen and not insured, necklace stolen but insured

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Quantitative problems

Example (Inventory)

Your football club needs to provide playing uniforms for each of its members. The initial order needs to be placed before registrations are complete; *i.e.*, before the final number of members has been determined. The initial (early) order incurs a fee of $\$500$ plus $\$20$ per uniform. Late orders incur an additional $\$300$ fee plus the usual $\$20$ per uniform. Uniforms are sold for $\$40$ each.

- *actions* ($\mathcal{A}$): Order quantity (q): $O_0, O_1, O_2, \dots, O_q, \dots$
- *events* ($\mathcal{S}$): Membership (m): $r_0, r_1, r_2, \dots, r_m, \dots$
- *outcomes* (Ω): Profit is some binary function of q and m, $f(q, m)$ $\dots$

Oil exploration (revisited)

Example (Oil exploration)

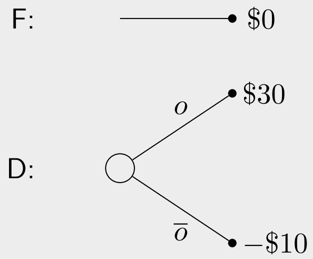
You're the chief petroleum engineer of an oil company which owns a drilling option on an area of sea. You're responsible for deciding whether or not to drill before the option expires.

- *actions* ($\mathcal{A}$): Drill (D), Forfeit rights (F) (don't drill ($\overline{\text{D}}$))
- *states* ($\mathcal{S}$): Oil present (o), no oil ($\overline{o}$)
- *outcomes* (Ω): Profit ($\$30$), loss ($-\$10$), status quo ($\$0$)

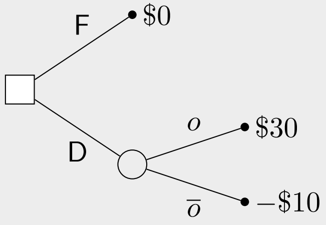
There is *uncertainty* due to incomplete information about *actual* state.

Oil exploration analysis

- Decide between two options:

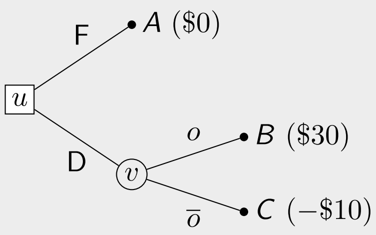


- Combined into a decision tree:



Choosing between uncertain situations is one of the fundamental problems of complex decision-making.

Decision trees



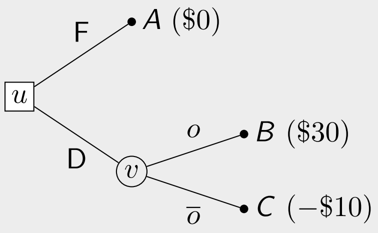
In a decision tree:

- each leaf node represents an outcome
- each branch represents either an action or an (chance) event
- internal nodes can be *decision nodes* (boxes) or *chance nodes* (circles)

Exercises

- What type of node is u? v? B?
- What does the branch labelled D represent?
- What does the branch labelled $\overline{o}$ represent?

Problem representation: decision tables



- Observation: Each combination of an action and a state uniquely determine an outcome
- Model as a binary function: $\omega : \mathcal{A} \times \mathcal{S} \to \Omega$

Represented as a table:

$\mathcal{S}$

	ω	o	$\overline{o}$
$\mathcal{A}$	F	A	A
	D	B	C

Decision tables:

- row = action
  column = state
- Interpretation: B $= \omega(\text{D}, o)$ means "B is the outcome of action D in state o";

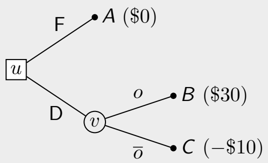
Decision tables

$\mathcal{S}$

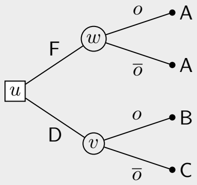
		s_1	s_2	$\dots$	s_k	$\dots$	s_n
$\mathcal{A}$	A_1	ω_{11}	ω_{12}	$\dots$	ω_{1k}	$\dots$	ω_{1n}
	A_2	ω_{21}	ω_{22}	$\dots$			
	$\vdots$						
	A_j	ω_{j1}	ω_{j2}	$\dots$	ω_{jk}	$\dots$	
	$\vdots$						
	A_m	ω_{m1}	ω_{m2}	$\dots$	ω_{mk}	$\dots$	ω_{mn}

- A decision table represents the binary function $\omega : \mathcal{A} \times \mathcal{S} \to \Omega$, where $\mathcal{A} = \{A_1, \dots, A_m\}$ and $\mathcal{S} = \{s_1, \dots, s_n\}$, and the entry in the j-th row and k-th column is $\omega_{jk} = \omega(A_j, s_k)$
- Formally, a 4-tuple $T = (\mathcal{A}, \Omega, \mathcal{S}, \omega)$

Trees and tables



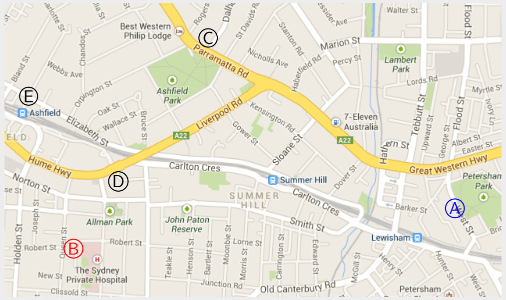
	o	$\overline{o}$
F	$0	$0
D	$30	−$10

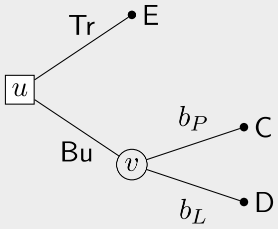


- Multiple trees may correspond to the same table
- Going from tables (*normal form*) to trees (*extensive form*) is straight forward, but the converse can be tricky
- Which representation is better: trees or tables?
- Which representation facilitates decision analysis most?

Trees

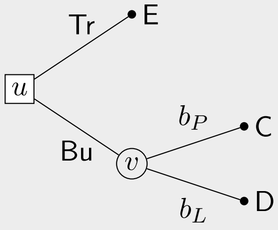
Problem representation: decision trees





- a *tree* is a *connected graph* with no *circuits/cycles*
- node connections are called *branches*
- a unique node may be designated as the tree's *root*; then we have a *rooted tree*

Tree definitions

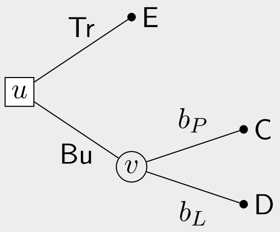


- a *path* is a sequence of nodes connected by branches
- the first node on a node's path to the root is called the node's *parent*; all other adjacent nodes are the node's *children*
- a node with no children is called a *leaf node*; a non-leaf node is called an *internal node*

Exercises

- Which nodes are the leaves? The internal nodes?
- Which nodes are the parents/children of node v? D? u?

Tree definitions



- a node u is an *ancestor* of node v if u lies on the path from the root to v (excluding v itself)
- the *descendants*, or *successors*, of a node v are all the nodes that have v as an ancestor
- The *subtree* with root v is the tree comprising only v and all its descendants

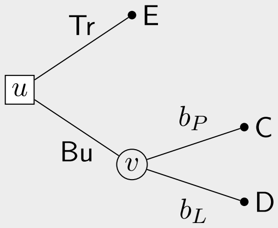
Exercises

- Which nodes are the ancestors of C? v? u?
- Which nodes are the descendants of E? v? u?
- Draw the subtrees of with respective roots: v, C, u

Tree properties

Theorem (Tree characterisation)

A graph is a tree if and only if there is a unique path between any two of its nodes.



Therefore, in a rooted tree:

- there is a unique path from every node to the root
- each node (except the root) has a unique parent, but may have multiple children

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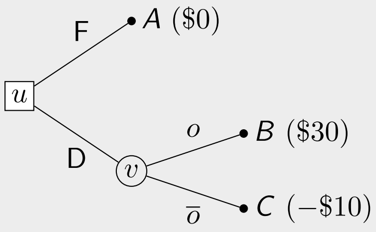
Decision trees



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Exercises

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Problem representation

Exercises

Draw decision trees for the problems below:

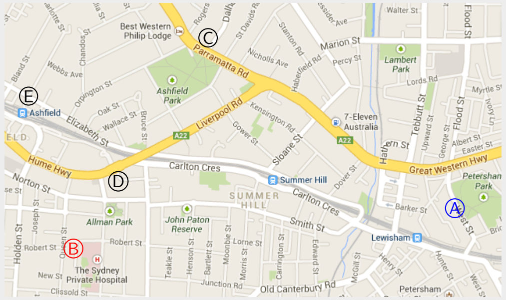
- Alice's insurance problem
- Alice's football club inventory problem

How would you modify the representations above if Alice had two insurance policies to choose from?

Outcomes

Comparing outcomes: value/payoff functions

- Preferences over outcomes can be easily expressed if the outcomes can be quantified numerically

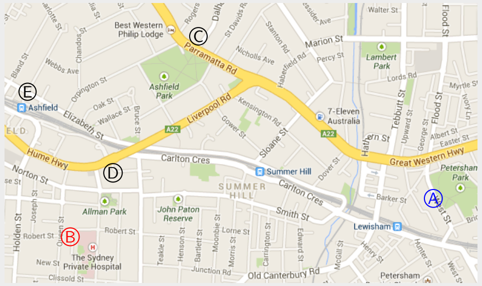


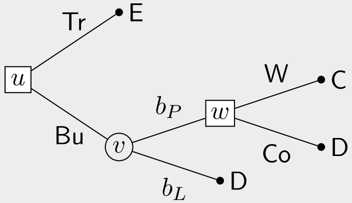
Distance to B:

ω	$d(\omega, \mathrm{B})$
B	0km
C	4km
D	1km
E	2km

- Prefer E to C because $d(\mathrm{E},\mathrm{B}) < d(\mathrm{C},\mathrm{B})$

Outcomes and values

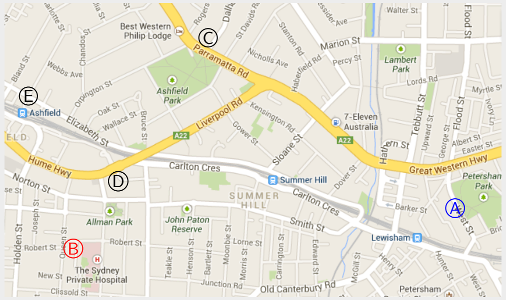




Question

Suppose that the route up Parramatta Rd loops around to D providing the new option to either walk from C or continue to D. Do the two D leaf nodes correspond to the same outcome if we evaluate them according to: (a) distance; (b) total travel time?

Outcomes and values



Values of outcomes based on distance (km):

	b_L	b_P
Tr	2	2
Bu	1	4

- Walking distance? Straight line?
- Consider values based on travel times (mins):

	b_L	b_P
Tr	30	30
Bu	10	40

Decision and preference

- Agents choose between outcomes according to their individual *preferences*
- One convenient way to encode preference is by assigning each outcome a numerical *value*; *e.g.*, money, distance, *etc*.
- Preferences are *subjective*: *i.e.*, each agent has its own *value function*: $v : \Omega \to \mathbb{R}$
- A value function is essentially a *random variable*
- A decision problem is now: $P = (\mathcal{A}, \mathcal{S}, \Omega, \omega, v)$

Covention

Value assignments usually assign higher values to more preferred (more desirable) outcomes.

The epistemic state

An agent's decisions depends on two main aspects:

- their preferences (*e.g.*, values on outcomes)
- their *epistemic state* (*e.g.*, information about the state at the world when the decision is made)

Definition (Epistemic state)

An agent's *epistemic state* is the knowledge (information) or belief it has about the actual state.

Decision problems and rules

Fundamental problem of decision theory

For any given decision problem, to come up with a *rational* choice from among the possible actions.

Definition (Decision rule)

A *decision rule* is a way of choosing, for each decision problem, an action or set of actions.

Questions:

- What constitutes a *rational* decision rule?
- How does an agent's epistemic state affect a decision rule?