THE UNIVERSITY OF NEW SOUTH WALES

SEMESTER 2 2017

COMP6741: PARAMETERIZED AND EXACT COMPUTATION - Trial Exam

1. TIME ALLOWED – 3 hours.
2. READING TIME – 10 minutes.
3. THIS EXAMINATION PAPER HAS 4 PAGES.
4. TOTAL NUMBER OF QUESTIONS – 7.
5. TOTAL MARKS AVAILABLE – 100.
6. THE QUESTIONS ARE NOT ALL OF EQUAL VALUE. MARKS AVAILABLE FOR EACH QUESTION ARE SHOWN IN THE EXAMINATION PAPER.
7. ALL ANSWERS MUST BE WRITTEN IN INK. EXCEPT WHERE THEY ARE EXPRESSLY REQUIRED, PENCILS MAY BE USED ONLY FOR DRAWING, SKETCHING OR GRAPHICAL WORK.
8. THIS PAPER MAY BE RETAINED BY THE CANDIDATE.

# SPECIAL INSTRUCTIONS

9. ANSWER ALL THE QUESTIONS.
10. CANDIDATES MAY BRING TO THE EXAMINATION: printed lecture notes, textbooks, handwritten and printed notes, UNSW approved calculator (but no other electronic devices).
11. THE FOLLOWING MATERIALS WILL BE PROVIDED: answer booklet.

Your answers may rely on theorems, lemmas and results stated in the lecture notes and exercise sheets of this course.

## 1 Kernel Lower Bound [10 marks]

Recall that a *clique* in a graph $G = (V, E)$ is a subset of vertices $S \subseteq V$ such that every two vertices from $S$ are adjacent in $G$. Consider the NP-complete Generalized Edge Clique Cover problem.

| Generalized Edge Clique Cover (GECC) | |
|---|---|
| Input: | A graph $G = (V, E)$, a subset of edges $R \subseteq E$, and an integer $k \leq \lvert V \rvert$ |
| Parameter: | $k$ |
| Question: | Is there a set $\mathcal{C}$ of at most $k$ cliques in $G$ such that each $e \in R$ is contained in at least one of these cliques? |

- Prove that Generalized Edge Clique Cover has no polynomial kernel unless coNP $\subseteq$ NP/poly.

## 2 ETH Lower Bound [10 marks]

A *triangle* in a graph is a clique of size 3, i.e., a set of 3 pairwise adjacent vertices. A *triangle transversal* in a graph $G = (V, E)$ is a subset of vertices $S \subseteq V$ such that $G - S$ has no triangle as a subgraph.

| Triangle Transversal (TT) | |
|---|---|
| Input: | A graph $G = (V, E)$ and an integer $k$ |
| Parameter: | $k$ |
| Question: | Does $G$ have a triangle transversal of size at most $k$? |

- Show that Triangle Transversal has no $2^{o(|V|)}$ time algorithm if the Exponential Time Hypothesis is true.

## 3 Treewidth [10 marks]

Consider the following two graph operations:

(i) Choose a vertex $v$ in the graph, and add a new vertex that is adjacent only to $v$.

(ii) Choose an edge $uv$ in the graph, remove the edge $uv$, and add a new vertex that is adjacent to $u$ and to $v$.

Let $G$ be a graph with treewidth at least 2. Let $H$ be a graph obtained from $G$ by a sequence of operations (i) and (ii).

- Show that the treewidth of $H$ is at most the treewidth of $G$.

# 4 Weighted Cycle Vertex Deletion [30 marks]

Consider the Weighted Cycle Vertex Deletion problem. A *cycle* is a 2-regular connected graph.

| Weighted Cycle Vertex Deletion | |
|---|---|
| Input: | Graph $G = (V, E)$, a weight function $\omega : V \to \mathbb{N}^+$ assigning an integer $\omega(v) \geq 1$ to every vertex $v \in V$, and an integer $k$ |
| Parameter: | $k$ |
| Question: | Is there a set $S \subseteq V$ with weight $\sum_{v \in S} \omega(v)$ at most $k$ such that $G - S$ is a cycle? |

1. Design simplification rules that transform $(G, \omega, k)$ into an equivalent instance $(G', \omega', k')$ such that
   (a) $G'$ has no vertex of degree at most 1, and [5 marks]
   (b) $G'$ has no degree-2 vertex with a neighbor of degree 2. [5 marks]
2. Show that a graph with minimum degree at least 2 and no two adjacent vertices of degree 2 has average degree at least 2.4. [10 marks]
3. Show that there is a (possibly randomized) algorithm for Weighted Cycle Vertex Deletion with running time $O^*(c^k)$ for some constant $c > 1$. [10 marks]

# 5 Feedback Vertex Set parameterized by vertex cover [10 marks]

Consider the following parameterization of the Feedback Vertex Set problem.

| vc-Feedback Vertex Set (vc-FVS) | |
|---|---|
| Input: | A graph $G = (V, E)$, an integer $k$, and a vertex cover $C$ of $G$ |
| Parameter: | $\lvert C \rvert$ |
| Question: | Does $G$ have a feedback vertex set of size at most $k$? |

- Show that vc-Feedback Vertex Set is fixed-parameter tractable.

# 6 W[1]-hardness [10 marks]

We denote by $G = (A \uplus B, E)$ a *bipartite graph* whose vertex set is partitioned into two independent sets $A$ and $B$. Consider the Hall Set problem, which asks for a subset $S$ of at most $k$ vertices in $A$ whose neighborhood in $B$ is smaller than $S$.

| Hall Set (HS) | |
|---|---|
| Input: | A bipartite graph $G = (A \uplus B, E)$ and an integer $k$ |
| Parameter: | $k$ |
| Question: | Is there a set $S \subseteq A$ of size at most $k$ such that $\lvert N(S) \rvert < \lvert S \rvert$? |

- Show that Hall Set is W[1]-hard.

**Hints**: Reduce from Clique. For a set $E'$ of edges, the set $V(E') = \{u \in e : e \in E'\}$ denotes the set of endpoints of $E'$. Observe that for a set $E'$ of $\binom{k}{2}$ edges, we have that $|V(E')| \leq k$ if and only if $V(E')$ is a clique of size $k$.

## 7 Local-Search-3-Sat [20 marks]

Consider the LOCAL-SEARCH-3-SAT problem.

| | |
|---|---|
| LOCAL-SEARCH-3-SAT (LS-3-SAT) | |
| Input: | A CNF formula $F$ where each clause contains at most 3 literals, an assignment $\alpha : \mathsf{var}(F) \to \{0,1\}$, and an integer $k$ |
| Parameter: | $k$ |
| Question: | Is there an assignment $\beta : \mathsf{var}(F) \to \{0,1\}$ that differs with $\alpha$ on at most $k$ variables and that satisfies $F$? |

1. Give a CNF formula $F$ on 3 variables and an assignment $\alpha$ such that $(F, \alpha, 2)$ is a NO-instance but $(F, \alpha, 3)$ is a YES-instance. [5 marks]
2. Design an $O^*(3^k)$ time algorithm for LOCAL-SEARCH-3-SAT. [10 marks]
3. Based on that algorithm, show that 3-SAT can be solved in $O^*(3^{n/2}) \subseteq O^*(1.7321^n)$ time, where $n = |\mathsf{var}(F)|$. [5 marks]

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