COMP4418: Knowledge Representation and Reasoning

Expressing Knowledge

Maurice Pagnucco
School of Computer Science and Engineering

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Knowledge engineering

KR is first and foremost about knowledge

- meaning and entailment
- find individuals and properties, then encode facts sufficient for entailments

Before implementing, need to understand clearly

- what is to be computed?
- why and where inference is necessary?

Example domain: university world

- people, lecturers, students, courses, graduations, awards, . . .

Task: KB with appropriate entailments

- what vocabulary?
- what facts to represent?

Vocabulary

Domain-dependent predicates and functions

- main question: what are the individuals?
- here: people, academics, students, courses, ...

named individuals

- alice, comp4418, facultyOfEngineering, foe, , ...

basic types

- Person, Academic, Student, Course, ...

attributes

- year1, year2, ..., core, elective, ...

relationships

- EnrolledIn, LecturerOf, ...

functions

- lecturerOf, licOf, bestFriendOf, ...

Basic facts

Usually atomic sentences and negations

- type facts
  Student(alice),
  Lecturer(barbara),
  Course(comp4418)
- property facts
  Difficult(comp4418),
  ¬Studious(allan),
  Studies(alice,comp4418)
- equality facts
  barbara = lecturerInCharge(comp1234),
  krr = comp4418,
  bestFriendOf(allan) = alice

Like a simple database
could store these facts in relational tables

Complex facts

Universal abbreviations

$\forall$ x. Lectures(lecturerInCharge(x), x))

$\forall$ x, y, z. (Lectures(x, y) $\wedge$ Studies(z, y)) $\rightarrow$ Teaches(x, z)

possible to express without quantifiers

Incomplete knowledge

Studies(alice, comp4418) $\vee$ Studies(allan, comp4418)

which?

stronger

$\forall x$. Studies(x, comp9444) $\vee$ Studies(x, comp9517)

$\exists x$[Student(x) $\wedge$ Studies(x, comp4418)]

who?

cannot write down more complete version

Closure axioms

$\forall x$[Student(x) $\rightarrow x =$ alice $\vee x =$ allan $\vee x =$ brad $\ldots$]

$\forall x \forall y$[Studies(x, y) $\rightarrow \ldots$]

$\forall x$[$x =$ comp4418 $\vee x =$ alice $\vee x =$ allan $\vee x =$ barbara $\ldots$]

limits domain of discourse

also useful to have alice$\neq$allan $\ldots$

Terminological facts

General relationships among predicates. For example:

- disjoint
  $\forall x[\text{Mammal}(x) \rightarrow \neg\text{Reptile}(x)]$
- subtype
  $\forall x[\text{Mammal}(x) \rightarrow \text{Animal}(x)]$
- exhaustive
  $\forall x[\text{Day}(x) \rightarrow \text{Monday}(x) \vee \ldots \vee \text{Sunday}(x)]$
- symmetry
  $\forall x \forall y[\text{RelatedTo}(x,y) \rightarrow \text{RelatedTo}(y,x)]$
- inverse
  $\forall x \forall y[\text{StudentOf}(x,y) \rightarrow \text{LecturerOf}(y,x)]$
- type restriction
  $\forall x \forall y[\text{Studies}(x,y) \rightarrow \text{Student}(x) \wedge \text{Course}(y)]$
- full definition
  $\forall x[\text{comp4418Student}(x) \equiv \text{Student}(x) \wedge \text{Studies}(x, \text{comp4418})]$
  $\forall x[\text{aiMajor}(x) \equiv \text{Student}(x) \wedge [(\text{Studies}(x, \text{comp4418}) \wedge \text{Studies}(x, \text{comp9444})) \vee (\text{Studies}(x, \text{comp4418}) \wedge \text{Studies}(x, \text{comp9517})) \vee (\text{Studies}(x, \text{comp9444}) \wedge \text{Studies}(x, \text{comp9517}))]]$
- Usually universally quantified conditionals or biconditionals

B&L (2005)

Entailments: 1

Is there a course whose Lecturer-in-Charge teaches Alice?

$\exists x[\text{Course}(x) \wedge \text{Teaches}(\text{lic}(x), \text{alice})]$??

Suppose $I \models KB$.

Then $I \models$ Course(comp4418)

Also $I \models \forall$ x. Lectures(lecturerInCharge(x), x))

so $I \models$ Lectures(lecturerInCharge(comp4418), comp4418).

Finally $I \models \forall$ x, y, z. (Lectures(x, y) $\wedge$ Studies(z, y)) $\rightarrow$ Teaches(x, z)

and $I \models$ Studies(alice, comp4418)

so $I \models$ Teaches(lecturerInCharge(comp4418), alice).

Thus, $I \models$ Course(comp4418) $\wedge$ Teaches(lecturerInCharge(comp4418), alice),

and so

$I \models \exists x[\text{Course}(x) \wedge \text{Teaches}(\text{lecturerInCharge}(x)), \text{alice})]$.

Can extract identity of Lecturer-in-Charge (since $I \models$ barbara = lecturerInCharge(comp4418))

Entailments: 2

If nobody is studying comp9444, then is there a someone studying comp9517 who is an AI major?
$\forall x[\text{Student}(x) \rightarrow \neg\text{Studies}(x, \text{comp9444})] \rightarrow \exists y[\text{Student}(y) \wedge \text{Studies}(y, \text{comp9517})]$??
Note: $KB \models (\alpha \rightarrow \beta)$ iff $KB \cup \{\alpha\} \models \beta$ (Deduction Theorem)
Assume: $I \models KB \cup \{\forall x[\text{Student}(x) \rightarrow \neg\text{Studies}(x, \text{comp9444})]\}$
Show: $I \models \exists y[\text{Student}(y) \wedge \text{Studies}(y, \text{comp9517}) \wedge \text{aiMajor}(y)]$

Have:	Student(alice)
and	$\forall x[\text{Student}(x) \rightarrow \neg\text{Studies}(x, \text{comp9444})]$
so	$\neg$Studies(alice, comp9444)
Also:	$\forall x.\ \text{Studies}(x, \text{comp9444}) \vee \text{Studies}(x, \text{comp9517})$
so	Studies(alice, comp9517)
Also:	Studies(alice, comp4418
Finally:	$\forall x[\text{aiMajor}(x) \equiv \text{Student}(x) \wedge [(\text{Studies}(x, \text{comp4418}) \wedge \text{Studies}(x, \text{comp9444})) \vee (\text{Studies}(x, \text{comp4418}) \wedge \text{Studies}(x, \text{comp9517})) \vee (\text{Studies}(x, \text{comp9444}) \wedge \text{Studies}(x, \text{comp9517}))]]$
so	aiMajor(alice)
Hence:	$\exists y[\text{Student}(y) \wedge \text{Studies}(y, \text{comp9517}) \wedge \text{aiMajor}(y)]$

Proof as sequence of sentences

What individuals?

Sometimes useful to reduce n-ary predicates to 1-place predicates and 1-place functions

- involves reifying properties: new individuals
- typical of description logics / frame languages (later)

Flexibility in terms of arity:

Purchases(john,sears,bike) or
Purchases(john,sears,bike,feb14) or
Purchases(john,sears,bike,feb14,$100)

Instead introduce purchase objects

Purchase(p) $\wedge$ agent(p)=john $\wedge$ obj(p)=bike $\wedge$ source(p)=sears $\wedge$ amount(p)=... $\wedge$ ...

allows purchase to be described at various levels of detail

Complex relationships:

MarriedTo(x,y) vs.
PreviouslyMarriedTo(x,y) vs.
ReMarriedTo(x,y)

Define marital status in terms of existence of marriages and divorces.

Marriage(m) $\wedge$ partner1(m)=x $\wedge$ partner2(m)=y $\wedge$ date(m)=... $\wedge$ witness(m)=... $\wedge$ ...

Abstract individuals

Also need individuals for numbers, dates, times, addresses, etc.

- objects about which we ask wh-questions

Quantities as individuals

age(suzy) = 14

age-in-years(suzy) = 14

age-in-months(suzy) = 168

perhaps better to have an object for the age of Suzy, whose value in years is 14

years(age(suzy)) = 14

months(x) = 12*years(x)

centimeters(x) = 100*meters(x)

Similarly with locations and times

instead of

time(m)="Jan 5 1992 4:47:03EST"

can use

time(m)=t $\wedge$ year(t)=1992 $\wedge$ ...

Other sorts of facts

Statistical / probabilistic facts

- Half of the companies are located on the East Side
- Most of the employees are restless
- Almost none of the employees are completely trustworthy

Default / prototypical facts

- Company presidents typically have secretaries intercepting their phone calls
- Cars have four wheels

Intentional facts

- John believes that Henry is trying to blackmail him
- Jane does not want Jim to think that she loves John

Others . . .