COMP4418: Knowledge Representation and Reasoning
Expressing Knowledge

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COMP4418, Week 2
Knowledge engineering

KR is first and foremost about knowledge

- meaning and entailment
- find individuals and properties, then encode facts sufficient for entailments

Before implementing, need to understand clearly

- what is to be computed?
- why and where inference is necessary?

Example domain: university world

- people, lecturers, students, courses, graduations, awards, ...

Task: KB with appropriate entailments

- what vocabulary?
- what facts to represent?

B&L (2005)
Vocabulary

Domain-dependent predicates and functions

- main question: what are the individuals?
- here: people, academics, students, courses, ...

named individuals

- alice, comp4418, facultyOfEngineering, foe, ...

basic types

- Person, Academic, Student, Course, ...

attributes

- year1, year2, ..., core, elective, ...

relationships

- EnrolledIn, LecturerOf, ...

functions

- lecturerOf, licOf, bestFriendOf, ...
Basic facts

Usually atomic sentences and negations

- **type facts**
  - Student(alice),
  - Lecturer(barbara),
  - Course(comp4418)
- **property facts**
  - Difficult(comp4418),
  - ¬Studious(allan),
  - Studies(alice,comp4418)
- **equality facts**
  - barbara = lecturerInCharge(comp1234),
  - krr = comp4418,
  - bestFriendOf(allan) = alice

Like a simple database
   could store these facts in relational tables

B&L (2005)
Complex facts

Universal abbreviations
\[ \forall x. \text{Lectures(lecturerInCharge(x), x)} \]
\[ \forall x, y, z. (\text{Lectures}(x, y) \land \text{Studies}(z, y)) \rightarrow \text{Teaches}(x, z) \]
possible to express without quantifiers

Incomplete knowledge
\[ \text{Studies}(alice, \text{comp4418}) \lor \text{Studies}(allan, \text{comp4418}) \]
which?
stronger
\[ \forall x. \text{Studies}(x, \text{comp9444}) \lor \text{Studies}(x, \text{comp9517}) \]
\[ \exists x[\text{Student}(x) \land \text{Studies}(x, \text{comp4418})] \]
who?
cannot write down more complete version

Closure axioms
\[ \forall x[\text{Student}(x) \rightarrow x = alice \lor x = allan \lor x = brad \ldots ] \]
\[ \forall x\forall y[\text{Studies}(x, y) \rightarrow \ldots ] \]
\[ \forall x[x = \text{comp4418} \lor x = alice \lor x = allan \lor x = barbara \ldots ] \]
limits domain of discourse
also useful to have alice \neq allan \ldots
Terminological facts

General relationships among predicates. For example:

- **disjoint**
  \[ \forall x [ \text{Mammal}(x) \rightarrow \neg \text{Reptile}(x)] \]

- **subtype**
  \[ \forall x [ \text{Mammal}(x) \rightarrow \text{Animal}(x)] \]

- **exhaustive**
  \[ \forall x [ \text{Day}(x) \rightarrow \text{Monday}(x) \lor \ldots \lor \text{Sunday}(x)] \]

- **symmetry**
  \[ \forall x \forall y [ \text{RelatedTo}(x, y) \rightarrow \text{RelatedTo}(y, x)] \]

- **inverse**
  \[ \forall x \forall y [ \text{StudentOf}(x, y) \rightarrow \text{LecturerOf}(y, x)] \]

- **type restriction**
  \[ \forall x \forall y [ \text{Studies}(x, y) \rightarrow \text{Student}(x) \land \text{Course}(y)] \]

- **full definition**
  \[ \forall x [ \text{comp4418Student}(x) \equiv \text{Student}(x) \land \text{Studies}(x, \text{comp4418})] \]
  \[ \forall x [ \text{aiMajor}(x) \equiv \text{Student}(x) \land [(\text{Studies}(x, \text{comp4418}) \land \text{Studies}(x, \text{comp9444})) \lor (\text{Studies}(x, \text{comp4418}) \land \text{Studies}(x, \text{comp9517})) \lor (\text{Studies}(x, \text{comp9444}) \land \text{Studies}(x, \text{comp9517}))]] \]

- Usually universally quantified conditionals or biconditionals
Is there a course whose Lecturer-in-Charge teaches Alice?

$\exists x [\text{Course}(x) \land \text{Teaches}(\text{lic}(x), \text{alice})]$ ??

Suppose $I \models KB$.

Then $I \models \text{Course}(\text{comp4418})$

Also $I \models \forall x. \text{Lectures}(\text{lecturerInCharge}(x), x))$

so $I \models \text{Lectures}(\text{lecturerInCharge}(\text{comp4418}), \text{comp4418})$.

Finally $I \models \forall x, y, z. (\text{Lectures}(x, y) \land \text{Studies}(z, y)) \rightarrow \text{Teaches}(x, z)$

and $I \models \text{Studies}(\text{alice}, \text{comp4418})$

so $I \models \text{Teaches}(\text{lecturerInCharge}(\text{comp4418}), \text{alice})$.

Thus, $I \models \text{Course}(\text{comp4418}) \land \text{Teaches}(\text{lecturerInCharge}(\text{comp4418}), \text{alice})$,

and so

$I \models \exists x [\text{Course}(x) \land \text{Teaches}(\text{lecturerInCharge}(x)), \text{alice})$.

Can extract identity of Lecturer-in-Charge (since $I \models \text{barbara} = \text{lecturerInCharge}(\text{comp4418})$ )
Entailments: 2

If nobody is studying comp9444, then is there a someone studying comp9517 who is an AI major?

∀x[Student(x) → ¬Studies(x, comp9444)] → ∃y[Student(y) ∧ Studies(y, comp9517)] ??

Note: KB |= (α → β) iff KB ∪ {α} |= β (Deduction Theorem)

Assume: I |= KB ∪ {∀x[Student(x) → ¬Studies(x, comp9444)]}

Show: I |= ∃y[Student(y) ∧ Studies(y, comp9517) ∧ aiMajor(y)]

Have: Student(alice)

and ∀x[Student(x) → ¬Studies(x, comp9444)]

so ¬Studies(alice, comp9444)

Also: ∀x. Studies(x, comp9444) ∨ Studies(x, comp9517)

so Studies(alice, comp9517)

Also: Studies(alice, comp4418)

Finally: ∀x[aiMajor(x) ≡ Student(x) ∧ [(Studies(x, comp4418) ∧ Studies(x, comp9444)) ∨ (Studies(x, comp4418) ∧ Studies(x, comp9517)) ∨ (Studies(x, comp9444) ∧ Studies(x, comp9517))]]

so aiMajor(alice)

Hence: ∃y[Student(y) ∧ Studies(y, comp9517) ∧ aiMajor(y)]

Proof as sequence of sentences
What individuals?

Sometimes useful to reduce n-ary predicates to 1-place predicates and 1-place functions

- involves reifying properties: new individuals
- typical of description logics / frame languages (later)

Flexibility in terms of arity:

Purchases(john, sears, bike) or
Purchases(john, sears, bike, feb14) or
Purchases(john, sears, bike, feb14, $100)

Instead introduce purchase objects

Purchase(p) \land \text{agent}(p)=\text{john} \land \text{obj}(p)=\text{bike} \land \text{source}(p)=\text{sears} \land \text{amount}(p)=\ldots \land \ldots

allows purchase to be described at various levels of detail

Complex relationships:

MarriedTo(x, y) vs.
PreviouslyMarriedTo(x, y) vs.
ReMarriedTo(x, y)

Define marital status in terms of existence of marriages and divorces.

Marriage(m) \land \text{partner1}(m)=x \land \text{partner2}(m)=y \land \text{date}(m)=\ldots \land \text{witness}(m)=\ldots \land \ldots
Abstract individuals

Also need individuals for numbers, dates, times, addresses, etc.

- objects about which we ask wh-questions

Quantities as individuals

age(suzy) = 14
age-in-years(suzy) = 14
age-in-months(suzy) = 168

perhaps better to have an object for the age of Suzy, whose value in years is 14

years(age(suzy)) = 14
months(x) = 12*years(x)
centimeters(x) = 100*meters(x)

Similarly with locations and times

instead of
time(m)=“Jan 5 1992 4:47:03EST”
can use
time(m)=t ∧ year(t)=1992 ∧ . . .
Other sorts of facts

Statistical / probabilistic facts
- Half of the companies are located on the East Side
- Most of the employees are restless
- Almost none of the employees are completely trustworthy

Default / prototypical facts
- Company presidents typically have secretaries intercepting their phone calls
- Cars have four wheels

Intentional facts
- John believes that Henry is trying to blackmail him
- Jane does not want Jim to think that she loves John

Others . . .