Exercise sheet 4

COMP6741: Parameterized and Exact Computation

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19T3

Exercise 1. A *Boolean formula in Conjunctive Normal Form (CNF)* is a conjunction (AND) of disjunctions (OR) of literals (a Boolean variable or its negation). A *HORN* formula is a CNF formula where each clause contains at most one positive literal. For a CNF formula F and an assignment $\tau : S \to \{0,1\}$ to a subset S of its variables, the formula $F[\tau]$ is obtained from F by removing each clause that contains a literal that evaluates to 1 under S, and removing all literals that evaluate to 0 from the remaining clauses.

HORN-Backdoor Detection	
Input:	A CNF formula F and an integer k.
Parameter:	k
Question:	Is there a subset S of the variables of F with $\|S\| \leq k$ such that for each assignment $\tau : S \to \{0,1\}$, the formula $F[\tau]$ is a HORN formula?

Example: $(\neg a \vee b \vee c) \wedge (b \vee \neg c \vee \neg d) \wedge (a \vee b \vee \neg e) \wedge (\neg b \vee c \vee \neg e)$ with $k = 1$ is a Yes-instance, certified by $S = \{b\}$.

- Show that HORN-Backdoor Detection is FPT using the fact that Vertex Cover is FPT.

Exercise 2. Show that Weighted Circuit Satisfiability $\in XP$.

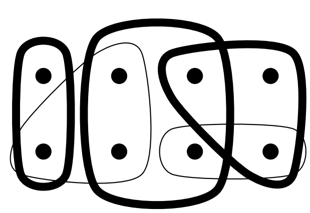
Exercise 3. Recall that a *k-coloring* of a graph $G = (V,E)$ is a function $f : V \to \{1,2,...,k\}$ assigning colors to V such that no two adjacent vertices receive the same color.

Multicolor Clique	
Input:	A graph $G = (V,E)$, an integer k, and a k-coloring of G
Parameter:	k
Question:	Does G have a clique of size k?

- Show that Multicolor Clique is W[1]-hard.

Exercise 4. A *set system* $\mathcal{S}$ is a pair (V,H), where V is a finite set of elements and H is a set of subsets of V. A *set cover* of a set system $\mathcal{S} = (V,H)$ is a subset X of H such that each element of V is contained in at least one of the sets in X, i.e., $\bigcup_{Y \in X} Y = V$.

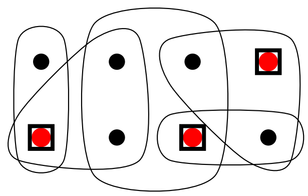
Set Cover	
Input:	A set system $\mathcal{S} = (V,H)$ and an integer k
Parameter:	k
Question:	Does $\mathcal{S}$ have a set cover of cardinality at most k?



- Show that SET COVER is W[2]-hard.

Exercise 5. A *hitting set* of a set system $\mathcal{S} = (V, H)$ is a subset X of V such that X contains at least one element of each set in H, i.e., $X \cap Y \neq \emptyset$ for each $Y \in H$.

HITTING SET	
Input:	A set system $\mathcal{S} = (V, H)$ and an integer k
Parameter:	k
Question:	Does $\mathcal{S}$ have a hitting set of size at most k?



- Show that HITTING SET is W[2]-hard.