Exercise sheet 4
COMP6741: Parameterized and Exact Computation
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19T3

Exercise 1. A Boolean formula in Conjunctive Normal Form (CNF) is a conjunction (AND) of disjunctions (OR) of literals (a Boolean variable or its negation). A HORN formula is a CNF formula where each clause contains at most one positive literal. For a CNF formula $F$ and an assignment $\tau : S \rightarrow \{0, 1\}$ to a subset $S$ of its variables, the formula $F[\tau]$ is obtained from $F$ by removing each clause that contains a literal that evaluates to 1 under $S$, and removing all literals that evaluate to 0 from the remaining clauses.

HORN-BACKDOOR DETECTION
Input: A CNF formula $F$ and an integer $k$.
Parameter: $k$
Question: Is there a subset $S$ of the variables of $F$ with $|S| \leq k$ such that for each assignment $\tau : S \rightarrow \{0, 1\}$, the formula $F[\tau]$ is a HORN formula?

Example: $(-a \lor b \lor c) \land (b \lor \neg c \lor \neg d) \land (a \lor b \lor \neg c) \land (\neg b \lor c \lor \neg e)$ with $k = 1$ is a Yes-instance, certified by $S = \{b\}$.

- Show that HORN-BACKDOOR DETECTION is FPT using the fact that VERTEX COVER is FPT.

Exercise 2. Show that WEIGHTED CIRCUIT SATISFIABILITY $\in$ XP.

Exercise 3. Recall that a $k$-coloring of a graph $G = (V, E)$ is a function $f : V \rightarrow \{1, 2, ..., k\}$ assigning colors to $V$ such that no two adjacent vertices receive the same color.

MULTICOLORED CLIQUE
Input: A graph $G = (V, E)$, an integer $k$, and a $k$-coloring of $G$
Parameter: $k$
Question: Does $G$ have a clique of size $k$?

- Show that MULTICOLORED CLIQUE is W[1]-hard.

Exercise 4. A set system $S$ is a pair $(V, H)$, where $V$ is a finite set of elements and $H$ is a set of subsets of $V$. A set cover of a set system $S = (V, H)$ is a subset $X$ of $H$ such that each element of $V$ is contained in at least one of the sets in $X$, i.e., $\bigcup_{Y \in X} Y = V$.

SET COVER
Input: A set system $S = (V, H)$ and an integer $k$
Parameter: $k$
Question: Does $S$ have a set cover of cardinality at most $k$?
• Show that Set Cover is W[2]-hard.

Exercise 5. A hitting set of a set system $S = (V, H)$ is a subset $X$ of $V$ such that $X$ contains at least one element of each set in $H$, i.e., $X \cap Y \neq \emptyset$ for each $Y \in H$.

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<th>Hitting Set</th>
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<td><strong>Input:</strong></td>
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• Show that Hitting Set is W[2]-hard.