# Search and Planning 

COMP3431 Robot Software Architectures

## So far ...

- Simple behaviour-based robots with no world model
- Robots that build models of space around them and remember events
- Robots that can use abstract models to answer questions and derive relations


## This time ...

- Using abstract representations to plan and solve problems


## Search

## Search

- Search is fundamental to Al
- We usually deal with problems that have no direct solution
- Must choose between alternatives


## State Transition Graph



## Problem Formulation

- Sets of states: description of one instant in time
- Initial state: where are we starting from?
- Actions: set of available actions
- Transition model: effect of each action
- Goal test: have we finished?
- Path cost: transitions may have different cost


## Search for Solutions

- Expand a node generating parent and child nodes
- Create frontier list (or open list)
- Search strategy: order in which frontier is expanded


## Example



After expanding Arad


After expanding Sibiu


## Uninformed Search

- Have no additional information beyond problem definition
- Depth-first
- Breadth-First


## Depth-First Search



## Breadth-First Search



## Depth-First Search

def dfs(start):
frontier = [start] while frontier ! = []:
first, rest $=$ frontier[0], frontier[1:] print("Node ", first.id)
frontier = first.neighbours + rest

## Breadth-First Search

def dfs(start):
frontier = [start] while frontier ! = []:
first, rest $=$ frontier[0], frontier[1:] print("Node ", first.id)
frontier $=$ rest + first.neighbours

## Iterative Deepening

- Depth-first to a limited depth
- Increase limit if goal not reached



## Informed (Heuristic) Search

- Use a heuristic (informed guess) to estimate cost to goal
- Greedy search expands nodes with lowest cost first


## Greedy Search



## A* Search

- Minimises total cost

$$
f(n)=g(n)+h(n)
$$

- $g$ is the cost to reach the node $n$, known exactly
- $h$ heuristic that guesses the cost to go form $n$ to the goal
- h is admissible if it never overestimates the cost


## A* Example



## History of $A^{*}$

- Invented for Shakey:

Hart, P. E., Nilsson, N. J. and Raphael, B. (1968). A Formal Basis for the Heuristic Determination of Minimum Cost Paths. IEEE Transactions on Systems Science and Cybernetics SSC4. 4 (2): 100-107.

- Shakey also gave rise to a planning framework that is still used


## Acknowledgments

- Examples from:

Russell, S. and Norvig, P. (2010). Artificial Intelligence: A Modern Approach (3rd Edition). Prentice Hall.

## Graph Search in Prolog



## Graph Representation

A graph may be represented by a set of edge predicates and a list of vertices.

```
edge(1, 5).
edge(2, 1).
edge(3, 1).
edge(4, 3).
edge(5, 8).
edge(6, 4).
edge(7, 5).
edge(8, 6).
```

```
edge(1, 7).
```

edge(1, 7).
edge(2, 7).
edge(2, 7).
edge(3, 6).
edge(3, 6).
edge(4, 5).
edge(4, 5).
edge(6, 5).
edge(8, 7).
vertices([1, 2, 3, 4, 5, 6, 7, 8]).

```

\section*{Finding a path}
- Write a program to find path from one node to another.
- Must avoid cycles (i.e. going around in circle).
- A template for the clause is:
path(Start, Finish, Visited, Path).

Start is the name of the starting node
Finish is the name of the finishing node
Visited is the list of nodes already visited.
Path is the list of nodes on the path, including Start and Finish.

\section*{The path program}
-The search for a path terminates when we have nowhere to go. path(Node, Node, _, [Node]).
- A path from Start to Finish starts with a node, X, connected to Start followed by a path from X to Finish.
```

path(Start, Finish, Visited, [Start | Path]) :-
edge(Start, X),
not(member(X, Visited)),
path(X, Finish, [X | Visited], Path).

```
member (X, [X|_]).
member (X, [_|Y]) :- member(X, Y).

\section*{Hamiltonian Paths}

A Hamiltonian path is a path which spans the entire graph without any repetition of nodes in the path.
```

hamiltonian(P) :-
vertices(V),
member(S, V),
path(S, _, [S], P),
subset(V, P).

```
:- hamiltonian(P).
\(P=[2,1,7,5,8,6,4,3]\)
\(P=[2,7,5,8,6,4,3,1]\)

\section*{Missionaries and Cannibals}
- There are three missionaries and three cannibals on the left bank of a river.
- They wish to cross over to the right bank using a boat that can only carry two at a time.
- The number of cannibals on either bank must never exceed the number of missionaries on the same bank, otherwise the missionaries will become the cannibals' dinner!
- Plan a sequence of crossings that will take everyone safely across.

\section*{}
- A state is one "snapshot" in time.
- For this problem, the only information we need to fully characterise the state is:
- the number of missionaries on the left bank,
- the number of cannibals on the left bank,
- the side the boat is on.
- All other information can be deduced from these three items.
- In Prolog, the state can be represented by a 3-arity term,
- state(Missionaries, Cannibals, Side)

\section*{}
- The solution consists of a list of moves, e.g.
```

[move(1, 1, right), move(2, 0, left)]

```
which we will take to mean that 1 missionary and 1 cannibal moved to the right bank, then 2 missionaries moved to the left bank.
- Like the graph search problem, we must avoid returning to a state we have visited before.
- The visited list will have the form:
```

[MostRecent State | ListOfPreviousStates]

```

\section*{Overview of Solution}
- We follow a simple graph search procedure:
- Start from an initial state
- Find a neighbouring state
- Check that the new state has not been visited before
- Find a path from the neighbour to the goal.
- The search terminates when we have found the state:
state(0, 0, right).

\section*{Top-level Prolog Code}
```

mandc(CurrentState, Visited, Path)
mandc(state(0, 0, right), _, []).
mandc(CurrentState, Visited, [Move | RestOfMoves]) :-
newstate(CurrentState, NextState),
not(member(NextState, Visited)),
make_move(CurrentState, NextState, Move),
mandc(NextState, [NextState | Visited], RestOfMoves).
make_move(state(M1, C1, left), state(M2, C2, right), move(M, C, right)) :-
M is M1-M2,
C is C1-C2.
make_move(state(M1, C1, right), state(M2, C2, left), move(M, C, left)) :-
M is M2 - M1,
C is C2-C1.

```

\section*{Possible Moves}
- A move is characterised by the number of missionaries and the number of cannibals taken in the boat at one time.
- Since the boat can carry no more than two people at once, the only possible combinations are:
\[
\begin{aligned}
& \text { carry }(2,0) . \\
& \text { carry }(1,0) \\
& \text { carry( } 1,1) \\
& \text { carry }(0,1) \\
& \text { carry }(0,2)
\end{aligned}
\]

Where carry(M, C) means the boat will carry M , missionaries and \(C\), cannibals on one trip.

\section*{Feasible Moves}
- Once we have found a possible move, we have to confirm that it is feasible.
- I.e. it is not feasible to move more missionaries or more cannibals than are present on one bank.
- When the state is state(M1, C1, left) and we try carry(M, C) then
\[
\mathrm{M}<=\mathrm{M} 1 \text { and } \mathrm{C}<=\mathrm{C} 1
\]
must be true.
- When the state is state(M1, C1, right) and we try carry(M, C) then
\[
\mathrm{M}+\mathrm{M} 1<=3 \text { and } \mathrm{C}+\mathrm{C} 1<=3
\]
must be true.

\section*{Legal Moves}
- Once we have found a feasible move, we must check that is legal.
- I.e. no missionaries must be eaten.
\[
\begin{aligned}
& \operatorname{legal}(\mathrm{X}, \mathrm{X}) \text { :- !. } \\
& \operatorname{legal}(3, \mathrm{X}):-\mathrm{C} \\
& \operatorname{legal}(0, \mathrm{x}) .
\end{aligned}
\]
- The only safe combinations are when there are equal numbers of missionaries and cannibals or all the missionaries are on one side.

\section*{Generating the next state}
```

newstate(state(M1, C1, left), state(M2, C2, right)) :-
carry(M, C),
M <= M1,
C <= C1,
M2 is M1 - M,
C2 is C1 - C,
legal(M2, C2).
newstate(state(M1, C1, right), state(M2, C2, left)) :-
carry(M, C),
M2 is M1 + M,
C2 is C1 + C,
M2 <= 3,
C2 <= 3,
legal(M2, C2).

```

\section*{Exercise - Constraint Satisfaction}

In five houses, each with a different colour, live five people of different nationalities, each of whom prefers a different brand of chocolates, a different drink, and a different pet. Given the following facts, answer the questions: "Where does the zebra live, and in which house do they drink water?"
- The Englishman lives in the red house.
- The Spaniard owns the dog.
- The Norwegian lives in the first house on the left.
- The green house is immediately to the right of the ivory house.
- The man who eats Cadburys lives in the house next to the man with the fox.
- Kit Kats are eaten in the yellow house.
- The Norwegian lives next to the blue house.
- The Smarties eater owns snails.
- The Snickers eater drinks orange juice.
- The Ukrainian drinks tea.
- The Japanese eats Milky Ways.
- Kit Kats are eaten in a house next to the house where the horse is kept.
- Coffee is drunk in the green house.
- Milk is drunk in the middle house.```

