Reasoning about Actions

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Reasoning about Actions

- McCarthy’s Advice Taker
  - Improve program behaviour by making statements to it
  - Program draws conclusions from its knowledge
    - Declarative conclusion: new knowledge
    - Imperative conclusion: take action
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- Actions change the environment, modify fluents
  - When you get on a bus, you are on the bus
  - When you get off a bus, you are not on the bus
  - When a bus moves, the position of the passengers changes
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- Want to model such environments
  - Action theory that models the actions and fluents
  - What does this theory entail?
Overview of the Lecture

- Three Problems
- The Situation Calculus
- Projection by regression
- Projection by progression
- Knowledge and sensing
- Concluding words
Three Problems

Commonsense problems, seemingly easy, yet very hard to formalise:

1. The Qualification Problem
2. The Frame Problem
3. The Ramification Problem
The Qualification Problem

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Represent the preconditions (qualifications) of an action.
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What must be true for this to be possible?

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  - Important qualification: $d$ is on $b$’s route
  - Minor qualification: fuel, driver, keys, …
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- Non-monotonic reasoning
  - Action is possible when all important qualifications hold, unless a minor qualification prevents it
  - Not specific to actions: a bird flies unless it’s abnormal
The Frame Problem

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Ex.: You don’t magically disappear from the bus when it moves. The weather also remains unchanged when the bus moves.
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**Ex.:** You don’t magically disappear from the bus when it moves. The weather also remains unchanged when the bus moves.

- Frame axioms specify what does *not* change
  - If you are on a bus, then you’re still on the bus when it moves.
  - If you are not on a bus, then you’re still not on the bus when it moves.
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  - If you are on a bus, then you’re still on the bus when it moves.
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- A actions, F fluents \( \Rightarrow \) about \( 2 \times A \times F \) frame axioms
  - 100 actions, 100 fluents \( \Rightarrow \) 20 000 frame axioms
  - Impractical to write down
  - Need to generate them or represent them implicitly
State Constraints

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Ex.: If you’re on the bus, your location is where the bus is. You cannot be at two busses at once.

- Indirect effect: action effects must adhere to state constraints
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- Indirect qualification: action allowed only if state constraint won’t be violated
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Ex.: If you’re on the bus, your location is where the bus is. You cannot be at two busses at once.

- Indirect effect: action effects must adhere to state constraints

- Indirect qualification: action allowed only if state constraint won’t be violated

- Constraints can often be compiled to qualifications, effects
  - When a bus moves, its passengers move along
  - You can get on a bus only if you’re not on a bus already
Our Approach (due to Ray Reiter)

We’ll focus on the **frame problem**.

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### The Frame Problem

Represent what is left unchanged by an action.

- Simple solution to the frame problem due to Reiter:
  
  \[ F \text{ holds after } a \iff a \text{ enables } F \text{ or } F \text{ holds before } a \text{ and } a \text{ does not disable } F \]

- Ignore the minor qualifications

- Compile state constraints to qualifications and effects

**Want:** a way to generate *frame axioms* from given effect axioms. **Why?**

- Modularity: could easily add new fluents / actions
- Accuracy: wouldn’t forget frame axioms
Overview of the Lecture

- Three Problems
- The Situation Calculus
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- Projection by progression
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- Concluding words
The Language of the Situation Calculus

**Terms** of two different **sorts**:

- Variables, standard names, functions of sort \( \{ \text{object} \} \)
- Action terms

For simplicity: non-nested functions, function only on the left-hand side

Special condition: action term

\[
\text{getOn}(M50) \quad \text{is a standard name}
\]

Then

\[
| = \text{getOn}(M50) \neq \text{getOff} \neq \text{goTo}(M50) \neq \ldots
\]

Formulas:

\[
P(t_1, \ldots, t_j)
\]

\[
t_1 = t_2 \neg \alpha \left( \alpha \lor \beta \right) \exists x \alpha
\]

- \([t]\) \alpha \alpha \text{ holds after action } t

- \(\square \alpha \alpha \text{ holds after any sequence of actions}\)

- \(\text{Poss}(t) \) represents precondition of action } t
The Language of the Situation Calculus

**Terms** of two different sorts:

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\[ t_1 = t_2 \neg \alpha \land \alpha \lor \beta \exists x \alpha \]
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**Ex.**: If M50 is an object standard name and getOn is an action function, then getOn(M50) is an action standard name.
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**Ex.**: If M50 is an object standard name and getOn is an action function, then getOn(M50) is an action standard name. Then \( \models \) getOn(M50) \( \neq \) getOff \( \neq \) goTo(M50, Uni) \( \neq \ldots \)
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**Ex.:** If M50 is an object standard name and getOn is an action function, then getOn(M50) is an action standard name. Then \( \models \text{getOn}(M50) \neq \text{getOff} \neq \text{goTo}(M50, \text{Uni}) \neq \ldots ! \)

**Formulas:**

- \( P(t_1, \ldots, t_j) \quad t_1 = t_2 \quad \neg \alpha \quad (\alpha \lor \beta) \quad \exists x \alpha \)
- \( [t] \alpha \quad \alpha \text{ holds after action } t \)
- \( \square \alpha \quad \alpha \text{ holds after any sequence of actions} \)
- Predicate Poss\((t)\) represents precondition of action \( t \)
Examples and Convention

- You don’t fall off the bus when the bus moves:
  \(\square (\forall b_1 \forall b_2 \forall d (\text{On}(b_1) \rightarrow [\text{goTo}(b_2, d)]\text{On}(b_1)))\)

- You cannot be on two busses at once:
  \(\square (\forall b_1 \forall b_2 (b_1 \neq b_2 \rightarrow \neg\text{On}(b_1) \lor \neg\text{On}(b_2)))\)

- \(F\) holds after \(a\) \(\iff\) \(a\) enables \(F\) or \(F\) holds before \(a\) and \(a\) does not disable \(F\)
  \(\square (\forall a \forall \vec{x} ([a]F(\vec{x}) \leftrightarrow \gamma^+ \lor (F(\vec{x}) \land \neg \gamma^-)))\)

Convention:

- \(\forall \vec{t}\) stands for \(\forall t_1 \ldots \forall t_j, F(\vec{t})\) for \(F(t_1, \ldots, t_j)\)
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Examples and Convention

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- \( F \) holds after \( a \) \iff \( a \) enables \( F \) or
  
  \( F \) holds before \( a \) and \( a \) does not disable \( F \)
  
  \[ \Box [a] F(\vec{x}) \leftrightarrow \gamma^+ \lor (F(\vec{x}) \land \neg \gamma^-) \]

Convection:

- \( \forall \vec{t} \) stands for \( \forall t_1 \ldots \forall t_j, F(\vec{t}) \) for \( F(t_1, \ldots, t_j) \)
- Operator \( \Box \) has maximum scope
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Examples and Convention

- You don’t fall off the bus when the bus moves:
  $\square \text{On}(b_1) \implies \lbrack \text{goTo}(b_2, d) \rbrack \text{On}(b_1)$

- You cannot be on two busses at once:
  $\square b_1 \neq b_2 \implies \neg \text{On}(b_1) \lor \neg \text{On}(b_2)$

- $F$ holds after $a \iff a$ enables $F$ or

  $F$ holds before $a$ and $a$ does not disable $F$

  $\square [a]F(x) \iff \gamma^+ \lor (F(x) \land \neg \gamma^-)$

Convention:

- $\forall \vec{t}$ stands for $\forall t_1 \ldots \forall t_j, F(t)$ for $F(t_1, \ldots, t_j)$

- Operator $\square$ has maximum scope

- Free variables are implicitly universally quantified

- We sometimes identify a (finite) set $\Sigma$ of sentences $\{\alpha_1, \ldots, \alpha_j\}$ with the conjunction $\alpha_1 \land \ldots \land \alpha_j$
Worlds and Situations

\[ w[\text{On}(M50), \langle \rangle] = 0 \]
\[ w[\text{pos}, \langle \rangle] = \text{Central} \]
\[ w[\text{On}(M50), \text{getOn}(M50)] = 1 \]
\[ w[\text{pos}, \text{getOn}(M50)] = \text{Central} \]
\[ w[\text{On}(M50), \text{getOn}(M50) \cdot \text{goTo}(M50, \text{Uni})] = 1 \]
\[ w[\text{pos}, \text{getOn}(M50) \cdot \text{goTo}(M50, \text{Uni})] = \text{Uni} \]
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Worlds and Situations (2)

Definition: situation, world

A **situation** \( z \) is a sequence of action standard names.
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A **situation** $z$ is a sequence of action standard names.

A **world** $w$ is a function that maps
- primitive functions $f(\vec{n})$ and situations to standard names, and
- primitive atomic formulas $P(\vec{n})$ and situations to $\{0, 1\}$. 
Worlds and Situations (2)

Definition: situation, world

A **situation** \( z \) is a sequence of action standard names.

A **world** \( w \) is a function that maps

- primitive functions \( f(\vec{n}) \) and situations to standard names, and
- primitive atomic formulas \( P(\vec{n}) \) and situations to \( \{0, 1\} \).

The **denotation** of a ground term w.r.t. \( w \) in \( z \) is defined as

- \( w(n, z) \overset{\text{def}}{=} n \) for every standard name \( n \)
- \( w(f(n_1, \ldots, n_j), z) \overset{\text{def}}{=} w[f(n_1, \ldots, n_j), z] \)

Recall: for simplicity we don’t consider nested functions, so \( f \) can only be applied to variables or names
The Semantics of the Situation Calculus

Definition: semantics

- \( w, z \models P(t_1, \ldots, t_j) \iff w[P(w(t_1, z), \ldots, w(t_j, z), z)] = 1 \)
- \( w, z \models t_1 = t_2 \iff w(t_1, z) = w(t_2, z) \)
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- \( w, z \models t_1 = t_2 \iff w(t_1, z) = w(t_2, z) \)
- \( w, z \models \neg \alpha \iff w, z \not\models \alpha \)
- \( w, z \models (\alpha \lor \beta) \iff w, z \models \alpha \text{ or } w, z \models \beta \)
- \( w, z \models \exists x \alpha \iff w, z \models \alpha^x_n \) for some std. name \( n \) of \( x \)’s sort
The Semantics of the Situation Calculus

### Definition: semantics

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- \( w, z \models [n] \alpha \iff w, z \cdot n \models \alpha \)

- \( w, z \models \Box \alpha \iff w, z \cdot z' \models \alpha \) for all situations \( z' \)

- \( \Sigma \models \alpha \iff \) for all \( w \), if \( w, \langle \rangle \models \beta \) for all \( \beta \in \Sigma \), then \( w, \langle \rangle \models \alpha \)
Example

\[ w \models \neg \text{On}(b) \]
\[ w \models [\text{getOn}(b)] \text{On}(b) \]
\[ w \models [\text{getOn}(b)][\text{goTo}(b, d)] \text{On}(b) \]
\[ w \models [\text{getOn}(b)][\text{goTo}(b, d)] \text{pos} = d \]
\[ w \models \exists a_1 \exists a_2 [a_1][a_2] \text{pos} = d \]
Solving the Frame Problem – Reiter’s Idea

When are we on a bus?
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Effect axioms:

□ [getOn(b)]On(b)
□ [getOff]¬On(b)
Solving the Frame Problem – Reiter’s Idea

When are we on a bus?

Effect axioms:

\[ \square a = \text{getOn}(b) \rightarrow [a] \text{On}(b) \]
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Solving the Frame Problem – Reiter’s Idea

When are we on a bus?

Effect axioms:

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\[ \square a = \text{getOff} \rightarrow [a]\neg\text{On}(b) \]

Assume causal completeness, i.e., assume:

\[ \square \neg\text{On}(b) \land [a] \quad \text{On}(b) \rightarrow a = \text{getOn}(b) \]
\[ \square \quad \text{On}(b) \land [a]\neg\text{On}(b) \rightarrow a = \text{getOff} \]
Solving the Frame Problem – Reiter’s Idea

When are we on a bus?

Effect axioms:

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So we get:

\[ \square [a]\text{On}(b) \leftrightarrow a = \text{getOn}(b) \lor (\text{On}(b) \land \neg a = \text{getOff}) \]

Done! This is called a **successor-state axiom**.

Proof on paper
Definition: successor-state axiom

A **successor-state axiom** has the form
\[ \Box [a]F(\vec{x}) \iff \gamma_F \]
or
\[ \Box [a]f(\vec{x}) = y \iff \gamma_f \]
where \( \gamma_F, \gamma_f \) do not mention \( \Box \) or \([t]\) operators.
**Definition: successor-state axiom**

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where $\gamma_F, \gamma_f$ do not mention $\Box$ or $[t]$ operators.

**Typical form of**

- $\gamma_F$ is $\gamma_F^+ \lor (F(\vec{x}) \land \neg \gamma_F^-)$
- $\gamma_f$ is $\gamma_f^+ \lor (f(\vec{x}) = y \land \neg \exists y' \gamma_f^+ y')$
**Successor-State Axioms**

**Definition: successor-state axiom**

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**Typical form of**

- \( \gamma_F \) is \( \gamma_F^+ \lor (F(\vec{x}) \land \neg \gamma_F^-) \)
- \( \gamma_f \) is \( \gamma_f^+ \lor (f(\vec{x}) = y \land \neg \exists y' \gamma_f^+ y') \)

Make sure that \( \models \gamma_f^{y_1} \land \gamma_f^{y_2} \rightarrow y_1 = y_2 \). Otherwise: inconsistency!
Examples

- You’re on a bus $\iff$ you got on it or you were on it and didn’t get off it:
  $\Box [a] \text{On}(b) \iff a = \text{getOn}(b) \lor (\text{On}(b) \land a \neq \text{getOff})$

- Your position is $p$ $\iff$ you were on a bus that moved to $p$ or you were at $p$ already and not on a bus that moved:
  $\Box [a] \text{pos} = p \iff \exists b (a = \text{goTo}(b, p) \land \text{On}(b)) \lor (\text{pos} = p \land \neg \exists d \exists b (a = \text{goTo}(b, d) \land \text{On}(b)))$
Basic Action Theories

An action theory must describe
- the initial situation
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- the initial situation
- how fluents change $\implies$ successor-state axioms
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- how fluents change $\implies$ successor-state axioms
- the action preconditions $\implies$ axiom for $\text{Poss}(a)$
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- how fluents change $\implies$ successor-state axioms
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**Definition: basic action theory**

$\Sigma_0 \land \Sigma_{\text{dyn}}$ is a **basic action theory** over a set of fluents $\mathcal{F}$ iff

- $\Sigma_{\text{dyn}}$ contains a successor-state axiom for every fluent in $\mathcal{F}$
- $\Sigma_{\text{dyn}}$ contains an axiom $\square \text{Poss}(a) \leftrightarrow \pi$
- $\Sigma_0, \pi$ mention no $\text{Poss}, \square, [t]$. 
Example: the Bus Scenario as Basic Action Theory

\[ a = \text{action}, \ b = \text{bus}, \ d = \text{destination}, \ p = \text{position} \]

- The initial situation:
  \[ \text{pos} = \text{Central} \land \text{Route(M50, Uni)} \]
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- **The initial situation:**
  \[\text{pos} = \text{Central} \land \text{Route(M50, Uni)}\]

- **You can get on/off a bus:**
  \[\Box [a] \text{On}(b) \iff a = \text{getOn}(b) \lor (\text{On}(b) \land a \neq \text{getOff})\]

- **You can move by being on a bus that moves:**
  \[\Box [a] \text{pos} = p \iff \exists b (a = \text{goTo}(b, p) \land \text{On}(b)) \lor \]
  \[\left( \text{pos} = p \land \neg \exists d \exists b (a = \text{goTo}(b, d) \land \text{On}(b)) \right)\]
Example: the Bus Scenario as Basic Action Theory

\( a = \text{action}, \ b = \text{bus}, \ d = \text{destination}, \ p = \text{position} \)

- The initial situation:
  \[
  \text{pos} = \text{Central} \land \text{Route(M50, Uni)}
  \]

- You can get on/off a bus:
  \[
  \square [a] \text{On}(b) \iff a = \text{getOn}(b) \lor (\text{On}(b) \land a \neq \text{getOff})
  \]

- You can move by being on a bus that moves:
  \[
  \square [a] \text{pos} = p \iff \exists b \ (a = \text{goTo}(b, p) \land \text{On}(b)) \lor
  (\text{pos} = p \land \neg \exists d \ \exists b \ (a = \text{goTo}(b, d) \land \text{On}(b)))
  \]

- You can’t get on (off) a bus when you’re on one (none), and a bus can only go along its route:
  \[
  \square \text{Poss}(a) \iff (\exists b \ a = \text{getOn}(b) \rightarrow \forall b \ \neg \text{On}(b)) \land
  (a = \text{getOff} \rightarrow \exists b \ \text{On}(b)) \land
  \forall b \forall d \ (a = \text{goTo}(b, d) \rightarrow \text{Route}(b, d))
  \]
The Projection Problem

The *central task* in reasoning about actions:

**Definition: projection problem**

Given a basic action theory:
Is a goal formula true in a future situation?

\[ \Sigma_0 \land \Sigma_{\text{dyn}} \models [t_1] \ldots [t_j] \alpha \]

**Want:** a way to *eliminate* \([t]\) operators.
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---

**Want:** a way to eliminate \([t]\) operators.

---

Two approaches:

- **Regression:** reduce to \(\Sigma_0 \models \alpha^*\)
- **Progression:** reduce to \(\Sigma_0^* \cup \Sigma_{\text{dyn}} \models \alpha\)
Overview of the Lecture

- Three Problems
- The Situation Calculus
- Projection by regression
- Projection by progression
- Knowledge and sensing
- Concluding words
Successor state axioms relate truth after $a$ to truth before $a$:

$\Box [a]F(\vec{x}) \leftrightarrow \gamma_F$, where $\gamma_F$ mentions no $[t]$
Successor state axioms relate truth after $a$ to truth before $a$:
$$\square [a] F(\vec{x}) \leftrightarrow \gamma_F,$$ where $\gamma_F$ mentions no $[t]$

Idea: successively replace $[r] F(\vec{t})$ with $\gamma_F^{\frac{a}{r}} \frac{x}{t}$
Regression – The Idea

- Successor state axioms relate truth after $a$ to truth before $a$:
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- Result: $\Sigma_0 \cup \Sigma_{\text{dyn}} \models [t_1] \ldots [t_j]\alpha$ reduces to $\Sigma_0 \cup \Sigma_{\text{dyn}} \models \alpha^*$
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Good: very simple and quite elegant
Successor state axioms relate truth after $a$ to truth before $a$:
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Good: very simple and quite elegant

Bad: $\alpha^*$ may grow exponentially
Regression

**Definition: regression operator, objective part**

Regression of $\alpha$ is defined w.r.t. a basic action theory where $\gamma_F, \gamma_f$ are the RHSs of the successor-state axioms and $\pi$ is the RHS of the Poss axiom. We assume no variable in $\alpha$ is quantified twice in the same scope (as in $\exists x (\alpha \lor \exists x \beta)$):

- $\mathcal{R}[z \cdot r, F(\overrightarrow{t})] \overset{\text{def}}{=} \mathcal{R}[z, \gamma_F a \overrightarrow{x}]$
- $\mathcal{R}[z \cdot r, f(\overrightarrow{t}) = t_0] \overset{\text{def}}{=} \mathcal{R}[z, \gamma_f a \overrightarrow{x} y]$ if $f$ is a function of sort object
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- $\mathcal{R}[, F(t)] \overset{\text{def}}{=} F(t)$
- $\mathcal{R}[, f(t) = t_0] \overset{\text{def}}{=} f(t) = t_0$ if $f$ is a function of sort object
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- $\mathcal{R}[z, \text{Poss}(t)] \overset{\text{def}}{=} \mathcal{R}[z, \pi^a_t]$
- $\mathcal{R}[z, (\alpha \vee \beta)] \overset{\text{def}}{=} (\mathcal{R}[z, \alpha] \vee \mathcal{R}[z, \beta])$
- $\mathcal{R}[z, \neg \alpha] \overset{\text{def}}{=} \neg \mathcal{R}[z, \alpha]$
- $\mathcal{R}[z, \exists x \alpha] \overset{\text{def}}{=} \exists x \mathcal{R}[z, \alpha]$
Regression

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- $\mathcal{R}[z, \exists x \alpha] \overset{\text{def}}{=} \exists x \mathcal{R}[z, \alpha]$
- $\mathcal{R}[z, [t] \alpha] \overset{\text{def}}{=} \mathcal{R}[z \cdot t, \alpha]$

The first parameter in $\mathcal{R}[z, \alpha]$ is the “situation stack”.

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**The Regression Result**

**Theorem: regression**

Let $\Sigma_0 \wedge \Sigma_{\text{dyn}}$ be a basic action theory over $\mathcal{F}$. Let $\alpha$ mention only fluents from $\mathcal{F} \cup \{\text{Poss}\}$ and no $\Box$.

$$\Sigma_0 \cup \Sigma_{\text{dyn}} \models \alpha \iff \Sigma_0 \models \mathcal{R}[\langle \rangle, \alpha]$$
Example

Let $\Sigma_0 \cup \Sigma_{\text{dyn}}$ be the bus scenario.

$$\Sigma_0 \cup \Sigma_{\text{dyn}} \models \left[ \text{getOn}(M50) \right] \left[ \text{goTo}(M50, \text{Uni}) \right] \text{pos} = \text{Uni} ?$$
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Example

\[ [a] \text{pos} = p \iff \exists b \ (a = \text{goTo}(b, p) \land \text{On}(b)) \lor \]
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\[ \iff \Sigma_0 \models R[\text{getOn}(M50) \cdot \text{goTo}(M50, \text{Uni}), \text{pos} = \text{Uni}] \]
\[ \iff \Sigma_0 \models R[\text{getOn}(M50), \gamma^{\text{pos} a}_{\text{goTo}(M50, \text{Uni})} \text{Uni}] \]
Example

\[ \square [a] \text{pos} = p \leftrightarrow \exists b \left( a = \text{goTo}(b, p) \land \text{On}(b) \right) \lor \left( \text{pos} = p \land \neg \exists d \exists b \left( a = \text{goTo}(b, d) \land \text{On}(b) \right) \right) \]

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\begin{align*}
\iff & \quad \Sigma_0 \models R[\langle \rangle, \left[ \text{getOn}(M50) \right] \left[ \text{goTo}(M50, \text{Uni}) \right] \text{pos} = \text{Uni}] \\
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\iff & \quad \Sigma_0 \models \exists b \left( \text{goTo}(M50, \text{Uni}) = \text{goTo}(b, \text{Uni}) \land R[\text{getOn}(M50), \text{On}(b)] \right) \lor \ldots
\end{align*}
Example

\[ \square [a] \text{On}(b) \leftrightarrow a = \text{getOn}(b) \lor (\text{On}(b) \land a \neq \text{getOff}) \]

Let \( \Sigma_0 \cup \Sigma_{\text{dyn}} \) be the bus scenario.

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\[ \Leftrightarrow \Sigma_0 \models \mathcal{R}[\langle \rangle, [\text{getOn}(M50)] [\text{goTo}(M50, \text{Uni})] \text{pos} = \text{Uni}] \]

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$$\left( \mathcal{R} [\langle \rangle, \text{On}(b)] \land \text{getOn}(M50) \neq \text{getOff} \right) \left) \lor \ldots \right)$$

$\iff$

$$\Sigma_0 \models \exists b \ (M50 = b \land (M50 = b \lor \mathcal{R} [\langle \rangle, \text{On}(b)])) \lor \ldots$$

Valid if $b$ is M50. So the whole formula is valid and hence entailed by $\Sigma_0$. 

$\checkmark$
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- Three Problems
- The Situation Calculus
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Progression – The Idea

- Want a new $\Sigma_0$ after action $t$
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- Idea: use $\gamma_F \frac{a}{r} \vec{x}^t$ to initialise new $F(t)$, forget old $F(t)$
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  - Second-order logic features quantification over predicates/functions
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    - $\text{goTo}(b, d)$ moves the passengers of the bus
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    - $\text{goTo}(b, d)$ moves the passengers of the bus
    - Indirect effects
  - Expressible subclasses are known
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Knowledge and Sensing

- New formulas: $K\alpha$  $O\alpha$
- Predicate $SF(t)$ represents sensing result of action $t$
Knowledge and Sensing

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- Predicate $SF(t)$ represents sensing result of action $t$

Ex.: You ask the driver whether the bus is going to UNSW
- “Yes” $\implies$ you know the bus going to UNSW
- “No” $\implies$ you know the bus is not going to UNSW
Knowledge and Sensing

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Formalisation of knowledge and sensing:
- Set of possible worlds $e$
- Doing $A$ tells you the value of $SF(A)$ in real world $w$
- Only consider those $w' \in e$ which agree with $w$
  
  If $w$ says bus goes to UNSW, only consider $w'$ where bus goes to UNSW
Definition: semantics of knowledge and sensing

\[ w \simeq_z w' \iff w, w' \text{ agree on the sensing results:} \]

- \( w \simeq \langle \rangle w' \)
- \( w \simeq_{z \cdot n} w' \iff w \simeq_z w' \text{ and } w[\text{SF}(n), z] = w'[\text{SF}(n), z] \)
The Semantics of Knowledge and Sensing

Definition: semantics of knowledge and sensing

\[ w \simeq_z w' \iff w, w' \text{ agree on the sensing results:} \]

- \[ w \simeq \langle \rangle w' \]
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An **epistemic state** \( e \) is a set of worlds.
Definition: semantics of knowledge and sensing

\( w \simeq_z w' \iff w, w' \) agree on the sensing results:

- \( w \simeq \langle \rangle w' \)
- \( w \simeq z \cdot n w' \iff w \simeq_z w' \) and \( w[SF(n), z] = w'[SF(n), z] \)

An **epistemic state** \( e \) is a set of worlds.

- Rules from Slide 14 retrofitted with additional \( e \) parameter,
  e.g., \( e, w, z \models \neg \alpha \iff e, w, z \not\models \alpha \)
The Semantics of Knowledge and Sensing

Definition: semantics of knowledge and sensing

\[ w \simeq_z w' \iff w, w' \text{ agree on the sensing results:} \]

- \( w \simeq \langle \rangle w' \)
- \( w \simeq_{z \cdot n} w' \iff w \simeq_z w' \text{ and } w[\text{SF}(n), z] = w'[\text{SF}(n), z] \)

An **epistemic state** \( e \) is a set of worlds.

- Rules from Slide 14 retrofitted with additional \( e \) parameter,
  e.g., \( e, w, z \models \neg \alpha \iff e, w, z \notmodels \alpha \)
- \( e, w, z \models \mathbf{K} \alpha \iff \text{for all worlds } w', \]
  \[ w' \in e \text{ and } w \simeq_z w' \Rightarrow e, w', z \models \alpha \]
The Semantics of Knowledge and Sensing

Definition: semantics of knowledge and sensing

\[ \approx_z w' \iff w, w' \text{ agree on the sensing results:} \]

\[ \begin{align*}
\text{w} \approx \emptyset \text{ w'} \\
\text{w} \approx_{z.n} \text{ w'} \iff \text{w} \approx_z \text{ w'} \text{ and } w[\text{SF}(n), z] = w'[\text{SF}(n), z]
\end{align*} \]

An epistemic state \( e \) is a set of worlds.

- Rules from Slide 14 retrofitted with additional \( e \) parameter, e.g., \( e, w, z \models \neg \alpha \iff e, w, z \not\models \alpha \)

- \( e, w, z \models K \alpha \iff \text{for all worlds } w' \),
  \[ w' \in e \text{ and } w \approx_z w' \Rightarrow e, w', z \models \alpha \]

- \( e, w, z \models O \alpha \iff \text{for all worlds } w' \),
  \[ w' \in e \text{ and } w \approx_z w' \Leftrightarrow e, w', z \models \alpha \]

\( \Sigma \models \alpha \iff \text{for all } e, w, \text{ if } e, w, \emptyset \models \beta \text{ for all } \beta \in \Sigma, \text{ then } e, w, \emptyset \models \alpha \)
Basic Action Theories with Knowledge

An action theory must describe

- what is true the initial situation
- what is \textit{known} about the initial situation
- how fluents change $\Rightarrow$ successor-state axioms
- the action preconditions $\Rightarrow$ axiom for $\text{Poss}(a)$
- how \textit{sensing} works $\Rightarrow$ axiom for $\text{SF}(a)$
Basic Action Theories with Knowledge

An action theory must describe

- what is true the initial situation
- what is *known* about the initial situation
- how fluents change $\implies$ successor-state axioms
- the action preconditions $\implies$ axiom for Poss($a$)
- how *sensing* works $\implies$ axiom for SF($a$)

**Definition: basic action theory**

$\Sigma_0 \land \Sigma_{dyn} \land O(\Sigma_1 \land \Sigma_{dyn})$ is a **basic action theory** over $\mathcal{F}$ iff

- $\Sigma_{dyn}$ contains a successor-state axiom for every fluent in $\mathcal{F}$
- $\Sigma_{dyn}$ contains an axiom $\Box$ Poss($a$) $\leftrightarrow \pi$
- $\Sigma_{dyn}$ contains an axiom $\Box$ SF($a$) $\leftrightarrow \varphi$
- $\Sigma_0, \Sigma_1, \pi, \varphi$ mention no Poss, SF, $\Box$, $[t]$. 
Example: the Bus Scenario as Basic Action Theory

- What is true, what is known initially:
  \[ \Sigma_0 \overset{\text{def}}{=} \text{pos} = \text{Central} \land \text{Route(M50, Uni)} \]
  \[ \Sigma_1 \overset{\text{def}}{=} \text{pos} = \text{Central} \]
Example: the Bus Scenario as Basic Action Theory

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  \[ \Sigma_1 \overset{\text{def}}{=} \text{pos} = \text{Central} \]

- \( \square [a] \text{On}(b) \leftrightarrow a = \text{getOn}(b) \lor (\text{On}(b) \land a \neq \text{getOff}) \)

- \( \square [a] \text{pos} = p \leftrightarrow \exists b \ (a = \text{goTo}(b, p) \land \text{On}(b)) \lor \]
  \[ (\text{pos} = p \land \neg \exists d \exists b \ (a = \text{goTo}(b, d) \land \text{On}(b))) \]

- \( \square \text{Poss}(a) \leftrightarrow (\exists b \ a = \text{getOn}(b) \rightarrow \forall b \neg \text{On}(b)) \land \]
  \[ (a = \text{getOff} \rightarrow \exists b \ \text{On}(b)) \land \]
  \[ \forall b \forall d \ (a = \text{goTo}(b, d) \rightarrow \text{Route}(b, d)) \]
Example: the Bus Scenario as Basic Action Theory

- What is true, what is known initially:
  \( \Sigma_0 \overset{\text{def}}{=} \text{pos} = \text{Central} \land \text{Route(M50, Uni)} \)
  \( \Sigma_1 \overset{\text{def}}{=} \text{pos} = \text{Central} \)

- \( \square [a] \text{On}(b) \iff a = \text{getOn}(b) \lor (\text{On}(b) \land a \neq \text{getOff}) \)

- \( \square [a] \text{pos} = p \iff \exists b \left( a = \text{goTo}(b, p) \land \text{On}(b) \right) \lor \left( \text{pos} = p \land \neg \exists d \exists b \left( a = \text{goTo}(b, d) \land \text{On}(b) \right) \right) \)

- \( \square \text{Poss}(a) \iff (\exists b \ a = \text{getOn}(b) \rightarrow \forall b \neg \text{On}(b)) \land \left( a = \text{getOff} \rightarrow \exists b \text{On}(b) \right) \land \forall b \forall d \left( a = \text{goTo}(b, d) \rightarrow \text{Route}(b, d) \right) \)

- You can ask and learn whether the bus stops at a destination:
  \( \square \text{SF}(a) \iff \forall b \forall d \left( a = \text{ask}(b, d) \rightarrow \text{Route}(b, d) \right) \)
Regression of Knowledge

Theorem: knowledge after action

\[ \models [a]K\alpha \iff (SF(a) \rightarrow K(SF(a) \rightarrow [a]\alpha)) \land \\
(\neg SF(a) \rightarrow K(\neg SF(a) \rightarrow [a]\alpha)) \]

Looks like a successor-state axiom, but it's a *theorem*!
Regression of Knowledge

Theorem: knowledge after action

\[ \models [a]K\alpha \iff (SF(a) \rightarrow K(SF(a) \rightarrow [a]\alpha)) \land \\
(\neg SF(a) \rightarrow K(\neg SF(a) \rightarrow [a]\alpha)) \]

Looks like a successor-state axiom, but it's a theorem!

Definition: regression operator, subjective part

- \( R[\langle \rangle, K \alpha] \overset{\text{def}}{=} KR[\langle \rangle, \alpha] \)
- \( R[z \cdot r, K \alpha] \overset{\text{def}}{=} R[z, (SF(r) \rightarrow K(SF(r) \rightarrow [r]\alpha))] \land \\
R[z, (\neg SF(r) \rightarrow K(\neg SF(r) \rightarrow [r]\alpha))] \]
- \( R[z, SF(t)] \overset{\text{def}}{=} R[z, \varphi_t^q] \)
The Regression Result with Knowledge

Theorem: regression

Let $\Sigma_0 \wedge \Sigma_{\text{dyn}} \wedge O(\Sigma_1 \wedge \Sigma_{\text{dyn}})$ be a basic action theory over $\mathcal{F}$.

Let $\alpha$ mention only fluents from $\mathcal{F} \cup \{\text{Poss}, \text{SF}\}$ and no $O$ or $\Box$.

$$
\Sigma_0 \wedge \Sigma_{\text{dyn}} \wedge O(\Sigma_1 \wedge \Sigma_{\text{dyn}}) \models \alpha \iff \Sigma_0 \wedge O\Sigma_1 \models \mathcal{R}[\langle \rangle, \alpha]
$$
The Regression Result with Knowledge

Theorem: regression

Let $\Sigma_0 \land \Sigma_{\text{dyn}} \land O(\Sigma_1 \land \Sigma_{\text{dyn}})$ be a basic action theory over $\mathcal{F}$. Let $\alpha$ mention only fluents from $\mathcal{F} \cup \{\text{Poss}, \text{SF}\}$ and no $O$ or $\Box$. Then:

$$\Sigma_0 \land \Sigma_{\text{dyn}} \land O(\Sigma_1 \land \Sigma_{\text{dyn}}) \models \alpha \iff \Sigma_0 \land O\Sigma_1 \models \mathcal{R}[\langle \rangle, \alpha]$$

Reasoning about actions + knowledge

+ Regression  (eliminates $[t]$)
+ Representation theorem  (eliminates $K$)

= Non-modal reasoning!
Overview of the Lecture

- Three Problems
- The Situation Calculus
- Projection by regression
- Projection by progression
- Knowledge and sensing
- Concluding words
Relationship to Planning

- Modelling dynamic systems is core AI
- In the beginning (1950ies, 1960ies): reasoning about action = planning
- McCarthy’s situation calculus (1963, 1969): too expressive, impractical
- Shakey introduced STRIPS for planning
- Reasoning about action and planning diverged
- Past years: they converge again
  - Reasoning action gets more efficient
  - Planning gets more expressive
  - Both sides benefit
Relevant Questions?

Reasoning about Knowledge
- Why not classical logic?

Semantics of knowledge
- How is $K\alpha$ defined?
- How is $O\alpha$ defined?
- How does quantification work?

Knowing that vs knowing what/who
- What’s the difference?
- Why is that semantic difference?

Representation theorem
- What are known instances?
- How does RES do it?

Logical Omniscience
- What does it mean?
- Why is it a problem?

Limited belief I
- Why more worlds?
- What is true/false support?
- When good/bad complexity?
- Why?

Limited belief II
- What’s unit propagation?
- What’s subsumption?
- How is $K_k\alpha$ defined?
- Soundness vs completeness?

Reasoning about actions
- What are the problems?

Solution of frame problem
- What’s a succ.-state axiom?
- What’s a basic action theory?

Projection
- What’s the projection task?
- What are the approaches?
- How does regression work?

Semantics of actions
- How are worlds defined?
- What does $SF(t)$ mean?
- How is $K\alpha$ defined in sitcalc?

This list is not intended to be exhaustive.