# Reasoning about Actions

Christoph Schwering

UNSW Sydney

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  - Improve program behaviour by making statements to it
  - Program draws conclusions from its knowledge
    - Declarative conclusion: new knowledge
    - Imperative conclusion: take **action**

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  - When you get off a bus, you are not on the bus
  - When a bus moves, the position of the passengers changes

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- Want to model such environments
  - Action theory that models the actions and fluents
  - What does this theory entail?

# Overview of the Lecture

- **Three Problems**
- The Situation Calculus
- Projection by regression
- Projection by progression
- Knowledge and sensing
- Concluding words

# Three Problems

Commonsense problems, seemingly easy, yet very hard to formalise:

1. The Qualification Problem
2. The Frame Problem
3. The Ramification Problem

# The Qualification Problem

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  - Minor qualification: fuel, driver, keys, ...
- Impractical to list all minor preconditions
- Non-monotonic reasoning
  - Action is possible when all important qualifications hold, unless a minor qualification prevents it
  - Not specific to actions: a bird flies unless it's abnormal

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  - If you are on a bus, then you're still on the bus when it moves.
  - If you are not on a bus, then you're still not on the bus when it moves.
- A actions, F fluents $\implies$ about $2 \times A \times F$ frame axioms
  - 100 actions, 100 fluents $\implies$ 20 000 frame axioms
  - Impractical to write down
  - Need to generate them or represent them implicitly

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- Indirect effect: action effects must adhere to state constraints
- Indirect qualification: action allowed only if state constraint won't be violated
- Constraints can often be compiled to qualifications, effects
  - When a bus moves, its passengers move along
  - You can get on a bus only if you're not on a bus already

# Our Approach (due to Ray Reiter)

We'll focus on the **frame problem**.

## The Frame Problem

Represent what is left unchanged by an action.

- Simple solution to the frame problem due to Reiter:
  $F$ holds after $a \iff a$ enables $F$ *or*
  $F$ holds before $a$ and $a$ does not disable $F$
- Ignore the minor qualifications
- Compile state constraints to qualifications and effects

**Want:** a way to *generate frame axioms* from given effect axioms. **Why?**

- Modularity: could easily add new fluents / actions
- Accuracy: wouldn't forget frame axioms

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**Terms** of two different **sorts**:

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Then $\models \mathrm{getOn}(\mathrm{M50}) \neq \mathrm{getOff} \neq \mathrm{goTo}(\mathrm{M50}, \mathrm{Uni}) \neq \ldots$!

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**Formulas**:

- $P(t_1, \ldots, t_j)$ $\quad t_1 = t_2$ $\quad \neg\alpha$ $\quad (\alpha \vee \beta)$ $\quad \exists x\, \alpha$
- $[t]\alpha$ — $\alpha$ holds after action $t$
- $\Box\, \alpha$ — $\alpha$ holds after any sequence of actions
- Predicate $\mathrm{Poss}(t)$ represents precondition of action $t$

# Examples and Convention

- You don't fall off the bus when the bus moves:
  $\Box\left(\forall b_1 \forall b_2 \forall d \left(\mathrm{On}(b_1) \rightarrow [\mathrm{goTo}(b_2, d)]\mathrm{On}(b_1)\right)\right)$

- You cannot be on two busses at once:
  $\Box\left(\forall b_1 \forall b_2 \left(b_1 \neq b_2 \rightarrow \neg\mathrm{On}(b_1) \vee \neg\mathrm{On}(b_2)\right)\right)$

- $F$ holds after $a$ $\iff$ $a$ enables $F$ *or*
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  $\Box\left(\forall a \forall \vec{x}\left([a]F(\vec{x}) \leftrightarrow \gamma^+ \vee (F(\vec{x}) \wedge \neg\gamma^-)\right)\right)$

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- Free variables are implicitly universally quantified
- We sometimes identify a (finite) set $\Sigma$ of sentences $\{\alpha_1, \ldots, \alpha_j\}$ with the conjunction $\alpha_1 \wedge \ldots \wedge \alpha_j$

# Worlds and Situations

$$w[\mathrm{On}(\mathrm{M50}),\ \langle\rangle] = 0$$
$$w[\mathrm{pos},\ \langle\rangle] = \mathrm{Central}$$
$$w[\mathrm{On}(\mathrm{M50}),\ \mathrm{getOn}(\mathrm{M50})] = 1$$
$$w[\mathrm{pos},\ \mathrm{getOn}(\mathrm{M50})] = \mathrm{Central}$$
$$w[\mathrm{On}(\mathrm{M50}),\ \mathrm{getOn}(\mathrm{M50}) \cdot \mathrm{goTo}(\mathrm{M50}, \mathrm{Uni})] = 1$$
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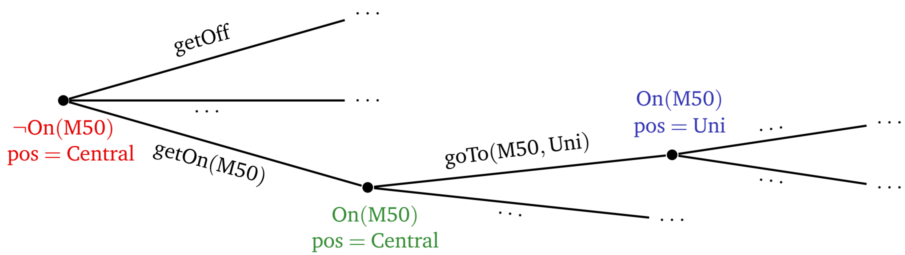
Tree view of $w$:



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- primitive functions $f(\vec{n})$ and situations to standard names, and
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The **denotation** of a ground term w.r.t. $w$ in $z$ is defined as

- $w(n, z) \stackrel{\text{def}}{=} n$ for every standard name $n$
- $w(f(n_1, \ldots, n_j), z) \stackrel{\text{def}}{=} w[f(n_1, \ldots, n_j), z]$

Recall: for simplicity we don't consider nested functions, so $f$ can only be applied to variables or names

# The Semantics of the Situation Calculus

## Definition: semantics

- $w, z \models P(t_1, \ldots, t_j) \iff w[P(w(t_1, z), \ldots, w(t_j, z), z] = 1$
- $w, z \models t_1 = t_2 \iff w(t_1, z) = w(t_2, z)$

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- $w, z \models \neg\alpha \iff w, z \not\models \alpha$
- $w, z \models (\alpha \vee \beta) \iff w, z \models \alpha$ or $w, z \models \beta$
- $w, z \models \exists x\, \alpha \iff w, z \models \alpha^x_n$ for some std. name $n$ of $x$'s sort

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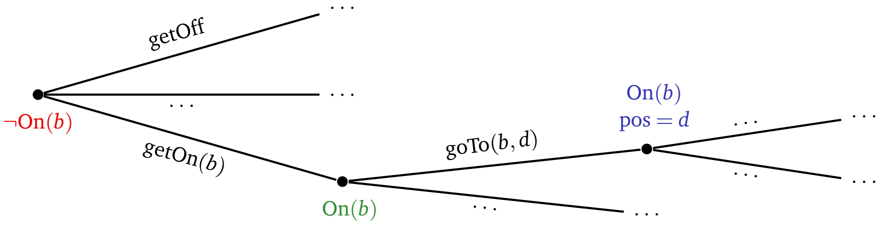
- $w, z \models [n]\alpha \iff w, z \cdot n \models \alpha$
- $w, z \models \Box\, \alpha \iff w, z \cdot z' \models \alpha$ for all situations $z'$

$\Sigma \models \alpha \iff$ for all $w$, if $w, \langle\rangle \models \beta$ for all $\beta \in \Sigma$, then $w, \langle\rangle \models \alpha$

# Example

$$w \models \neg \mathrm{On}(b)$$
$$w \models [\mathrm{getOn}(b)]\mathrm{On}(b)$$
$$w \models [\mathrm{getOn}(b)][\mathrm{goTo}(b,d)]\mathrm{On}(b)$$
$$w \models [\mathrm{getOn}(b)][\mathrm{goTo}(b,d)]\mathrm{pos} = d$$
$$w \models \exists a_1 \exists a_2 [a_1][a_2]\mathrm{pos} = d$$



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Assume **causal completeness**, i.e., assume:

$$\Box\, \neg\text{On}(b) \wedge [a]\ \ \text{On}(b) \to a = \text{getOn}(b)$$

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So we get:

$$\Box\, [a]\mathrm{On}(b) \leftrightarrow a = \mathrm{getOn}(b) \vee (\mathrm{On}(b) \wedge \neg a = \mathrm{getOff})$$

Done! This is called a **successor-state axiom**.

Proof on paper

# Successor-State Axioms

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A **successor-state axiom** has the form

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Typical form of

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Make sure that $\models \gamma_f{}^{y}_{y_1} \wedge \gamma_f{}^{y}_{y_2} \rightarrow y_1 = y_2$. Otherwise: inconsistency!

# Examples

- You're on a bus $\iff$ you got on it *or* you were on it and didn't get off it:

  $$\Box\,[a]\mathrm{On}(b) \leftrightarrow a = \mathrm{getOn}(b) \vee (\mathrm{On}(b) \wedge a \neq \mathrm{getOff})$$

- Your position is $p$ $\iff$ you were on a bus that moved to $p$ *or* you were at $p$ already and not on a bus that moved:

  $$\Box\,[a]\mathrm{pos} = p \leftrightarrow \exists b\,\big(a = \mathrm{goTo}(b,p) \wedge \mathrm{On}(b)\big) \vee \big(\mathrm{pos} = p \wedge \neg\exists d\,\exists b\,(a = \mathrm{goTo}(b,d) \wedge \mathrm{On}(b))\big)$$

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## Definition: basic action theory

$\Sigma_0 \wedge \Sigma_{\mathrm{dyn}}$ is a **basic action theory** over a set of fluents $\mathcal{F}$ iff

- $\Sigma_{\mathrm{dyn}}$ contains a successor-state axiom for every fluent in $\mathcal{F}$
- $\Sigma_{\mathrm{dyn}}$ contains an axiom $\Box\,\mathrm{Poss}(a) \leftrightarrow \pi$
- $\Sigma_0, \pi$ mention no $\mathrm{Poss}$, $\Box$, $[t]$.

# Example: the Bus Scenario as Basic Action Theory

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- You can move by being on a bus that moves:

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# Example: the Bus Scenario as Basic Action Theory

$a$ = action, $b$ = bus, $d$ = destination, $p$ = position

- The initial situation:

  $$\mathrm{pos} = \mathrm{Central} \wedge \mathrm{Route}(\mathrm{M50}, \mathrm{Uni})$$

- You can get on/off a bus:

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- You can't get on (off) a bus when you're on one (none), and a bus can only go along its route:

  $$\begin{aligned}\Box\, \mathrm{Poss}(a) \leftrightarrow\ & \left(\exists b\, a = \mathrm{getOn}(b) \rightarrow \forall b\, \neg \mathrm{On}(b)\right) \wedge \\ & \left(a = \mathrm{getOff} \rightarrow \exists b\, \mathrm{On}(b)\right) \wedge \\ & \forall b \forall d \left(a = \mathrm{goTo}(b, d) \rightarrow \mathrm{Route}(b, d)\right)\end{aligned}$$

# The Projection Problem

The *central task* in reasoning about actions:

## Definition: projection problem

Given a basic action theory:
Is a goal formula true in a future situation?

$$\Sigma_0 \land \Sigma_{\mathrm{dyn}} \models [t_1] \ldots [t_j]\alpha$$

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Two approaches:

- Regression: reduce to $\Sigma_0 \models \alpha^*$
- Progression: reduce to $\Sigma_0^* \cup \Sigma_{\text{dyn}} \models \alpha$

# Overview of the Lecture

- Three Problems
- The Situation Calculus
- **Projection by regression**
- Projection by progression
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- Concluding words

# Regression – The Idea

- Successor state axioms relate truth after $a$ to truth before $a$:
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- Good: very simple and quite elegant
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## Definition: regression operator, objective part

Regression of $\alpha$ is defined w.r.t. a basic action theory where $\gamma_F, \gamma_f$ are the RHSs of the successor-state axioms and $\pi$ is the RHS of the Poss axiom. We assume no variable in $\alpha$ is quantified twice in the same scope (as in $\exists x\,(\alpha \vee \exists x\,\beta)$):

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- $\mathcal{R}[z, [t]\alpha] \stackrel{\text{def}}{=} \mathcal{R}[z \cdot t, \alpha]$

The first parameter in $\mathcal{R}[z, \alpha]$ is the “situation stack”.

# The Regression Result

**Theorem: regression**

Let $\Sigma_0 \wedge \Sigma_{\text{dyn}}$ be a basic action theory over $\mathcal{F}$.
Let $\alpha$ mention only fluents from $\mathcal{F} \cup \{\text{Poss}\}$ and no $\square$.

$$\Sigma_0 \cup \Sigma_{\text{dyn}} \models \alpha \iff \Sigma_0 \models \mathcal{R}[\langle\rangle, \alpha]$$

# Example

Let $\Sigma_0 \cup \Sigma_{\mathrm{dyn}}$ be the bus scenario.

$\Sigma_0 \cup \Sigma_{\mathrm{dyn}} \models [\mathrm{getOn}(\mathrm{M50})][\mathrm{goTo}(\mathrm{M50}, \mathrm{Uni})]\mathrm{pos} = \mathrm{Uni}$ **?**

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$$\Box\,[a]\mathrm{pos} = p \leftrightarrow \exists b\,\big(a = \mathrm{goTo}(b,p) \land \mathrm{On}(b)\big) \lor \big(\mathrm{pos} = p \land \neg\exists d\,\exists b\,(a = \mathrm{goTo}(b,d) \land \mathrm{On}(b))\big)$$

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Formalisation of knowledge and sensing:

- Set of possible worlds $e$
- Doing $A$ tells you the value of $\mathrm{SF}(A)$ in real world $w$
- Only consider those $w' \in e$ which agree with $w$

  If $w$ says bus goes to UNSW, only consider $w'$ where bus goes to UNSW

# The Semantics of Knowledge and Sensing

## Definition: semantics of knowledge and sensing

$w \simeq_z w' \iff w, w'$ agree on the sensing results:

- $w \simeq_{\langle\rangle} w'$
- $w \simeq_{z \cdot n} w' \iff w \simeq_z w'$ and $w[\mathrm{SF}(n), z] = w'[\mathrm{SF}(n), z]$

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- $e, w, z \models \mathbf{O}\alpha \iff$ for all worlds $w'$,
  $w' \in e$ and $w \simeq_z w' \Leftrightarrow e, w', z \models \alpha$

$\Sigma \models \alpha \iff$ for all $e, w$, if $e, w, \langle\rangle \models \beta$ for all $\beta \in \Sigma$, then $e, w, \langle\rangle \models \alpha$

# Basic Action Theories with Knowledge

An action theory must describe

- what is true the initial situation
- what is *known* about the initial situation
- how fluents change $\implies$ successor-state axioms
- the action preconditions $\implies$ axiom for $\mathrm{Poss}(a)$
- how *sensing* works $\implies$ axiom for $\mathrm{SF}(a)$

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## Definition: basic action theory

$\Sigma_0 \wedge \Sigma_{\mathrm{dyn}} \wedge \mathbf{O}(\Sigma_1 \wedge \Sigma_{\mathrm{dyn}})$ is a **basic action theory** over $\mathcal{F}$ iff

- $\Sigma_{\mathrm{dyn}}$ contains a successor-state axiom for every fluent in $\mathcal{F}$
- $\Sigma_{\mathrm{dyn}}$ contains an axiom $\Box\,\mathrm{Poss}(a) \leftrightarrow \pi$
- $\Sigma_{\mathrm{dyn}}$ contains an axiom $\Box\,\mathrm{SF}(a) \leftrightarrow \varphi$
- $\Sigma_0, \Sigma_1, \pi, \varphi$ mention no $\mathrm{Poss}$, $\mathrm{SF}$, $\Box$, $[t]$.

# Example: the Bus Scenario as Basic Action Theory

- What is true, what is known initially:

$$\Sigma_0 \stackrel{\text{def}}{=} \text{pos} = \text{Central} \land \text{Route}(\text{M50}, \text{Uni})$$

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- $$\Box\, \text{Poss}(a) \leftrightarrow \left(\exists b\, a = \text{getOn}(b) \rightarrow \forall b\, \neg\text{On}(b)\right) \wedge \left(a = \text{getOff} \rightarrow \exists b\, \text{On}(b)\right) \wedge \forall b \forall d \left(a = \text{goTo}(b, d) \rightarrow \text{Route}(b, d)\right)$$

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- You can ask and learn whether the bus stops at a destination:

  $\Box\, \text{SF}(a) \leftrightarrow \forall b \forall d \left(a = \text{ask}(b, d) \rightarrow \text{Route}(b, d)\right)$

# Regression of Knowledge

**Theorem: knowledge after action**

$$\models [a]\mathbf{K}\alpha \leftrightarrow (\mathrm{SF}(a) \rightarrow \mathbf{K}(\mathrm{SF}(a) \rightarrow [a]\alpha)) \wedge$$
$$(\neg\mathrm{SF}(a) \rightarrow \mathbf{K}(\neg\mathrm{SF}(a) \rightarrow [a]\alpha))$$

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**Definition: regression operator, subjective part**

- $\mathcal{R}[\langle\rangle, \mathbf{K}\alpha] \stackrel{\text{def}}{=} \mathbf{K}\mathcal{R}[\langle\rangle, \alpha]$
- $\mathcal{R}[z \cdot r, \mathbf{K}\alpha] \stackrel{\text{def}}{=} \mathcal{R}[z, \big(\mathrm{SF}(r) \rightarrow \mathbf{K}(\mathrm{SF}(r) \rightarrow [r]\alpha)\big)] \wedge$
  $\mathcal{R}[z, \big(\neg\mathrm{SF}(r) \rightarrow \mathbf{K}(\neg\mathrm{SF}(r) \rightarrow [r]\alpha)\big)]$
- $\mathcal{R}[z, \mathrm{SF}(t)] \stackrel{\text{def}}{=} \mathcal{R}[z, \varphi_t^a]$

# The Regression Result with Knowledge

**Theorem: regression**

Let $\Sigma_0 \land \Sigma_{\mathrm{dyn}} \land \mathbf{O}(\Sigma_1 \land \Sigma_{\mathrm{dyn}})$ be a basic action theory over $\mathcal{F}$.
Let $\alpha$ mention only fluents from $\mathcal{F} \cup \{\mathrm{Poss}, \mathrm{SF}\}$ and no $\mathbf{O}$ or $\Box$.

$$\Sigma_0 \land \Sigma_{\mathrm{dyn}} \land \mathbf{O}(\Sigma_1 \land \Sigma_{\mathrm{dyn}}) \models \alpha \iff \Sigma_0 \land \mathbf{O}\Sigma_1 \models \mathcal{R}[\langle\rangle, \alpha]$$

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Reasoning about actions + knowledge

+ Regression (eliminates $[t]$)
+ Representation theorem (eliminates $\mathbf{K}$)

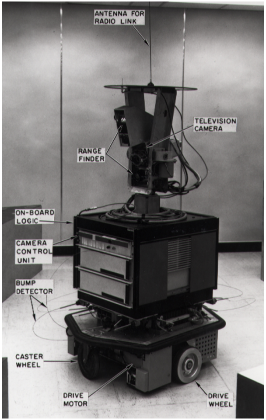
= Non-modal reasoning!

# Overview of the Lecture

- Three Problems
- The Situation Calculus
- Projection by regression
- Projection by progression
- Knowledge and sensing
- **Concluding words**

# Relationship to Planning

- Modelling dynamic systems is core AI
- In the beginning (1950ies, 1960ies):
  reasoning about action = planning
- McCarthy's situation calculus (1963, 1969):
  too expressive, impractical
- Shakey introduced STRIPS for planning
- Reasoning about action and planning diverged
- Past years: they converge again
  - Reasoning action gets more efficient
  - Planning gets more expressive
  - Both sides benefit



# Relevant Questions?

**Reasoning about Knowledge**

- Why not classical logic?

**Semantics of knowledge**

- How is $\mathbf{K}\alpha$ defined?
- How is $\mathbf{O}\alpha$ defined?
- How does quantification work?

**Knowing that vs knowing what/who**

- What's the difference?
- Why is that semantic difference?

**Representation theorem**

- What are known instances?
- How does RES do it?

**Logical Omniscience**

- What does it mean?
- Why is it a problem?

**Limited belief I**

- Why more worlds?
- What is true/false support?
- When good/bad complexity?
- Why?

**Limited belief II**

- What's unit propagation?
- What's subsumption?
- How is $\mathbf{K}_k\alpha$ defined?
- Soundness vs completeness?

**Implementation**

- How does DPLL work?
- Idea behind watched lits?
- Idea behind CDCL?

**Reasoning about actions**

- What are the problems?

**Solution of frame problem**

- What's a succ.-state axiom?
- What's a basic action theory?

**Projection**

- What's the projection task?
- What are the approaches?
- How does regression work?

**Semantics of actions**

- How are worlds defined?
- What does $\mathrm{SF}(t)$ mean?
- How is $\mathbf{K}\alpha$ defined in sitcalc?

This list is not intended to be exhaustive.