COMP9334 Solution to the tutorial for Week 10

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1. (a) Let us define a few notation:
   - $T_{\max}$ = computation time limit = 4000s
   - P = total number of cycles required = 10^7 Mcycles.
   - p_i = Speed (in Mcycles/s) of company i, e.g. $p_1 = 1000$ Mcycles/s
   - c_i = Per-second charge for company i, e.g. $c_1 = \$0.1$/s.
   - s_i = Set up cost for company i, e.g. $s_1 = 500$.

   The decision variables are y_i and x_i (for $i = 1, 2, 3$):

$$\begin{aligned} y_i &= \begin{cases} 1 & \text{if Company } i \text{ is chosen} \\ 0 & \text{otherwise} \end{cases} \\ x_i &= \text{fraction of cycles to be bought from Company } i \text{ and is } \geq 0 \end{aligned}$$

   Based on these decision variables, we know that
   - Completion time T is:

$$T = \max_i \frac{Px_i}{p_i}$$

   - Total cost C is:

$$C = \sum_{i=1}^{n} \left(\frac{c_i P x_i}{p_i} + s_i y_i \right)$$

   The problem formulation is

$$\begin{aligned} &\min \quad C \\ &\text{subject to} \\ &\quad T_{\max} \geq \frac{Px_i}{p_i} \qquad 1 \leq i \leq n \\ &\quad C = \sum_{i=1}^{n} \left(\frac{c_i P x_i}{p_i} + s_i y_i \right) \\ &\quad x_i \leq y_i \qquad 1 \leq i \leq n \\ &\quad \sum_{i=1}^{3} x_i = 1 \\ &\quad x_i \geq 0 \qquad 1 \leq i \leq n \\ &\quad y_i \in \{0, 1\} \qquad 1 \leq i \leq n \end{aligned}$$

(b) By using an integer programming solver, we find that we should buy 8×10^6 Mcycles from Company 2 and 2×10^6 Mcycles from Company 3. This costs a total of \$1833.

The files used to solve this problem are `hw_grid.mod`, `hw_grid.dat` and `hw_grid_batch`.

2. (a) Since Database 1 can only be placed at one of the three locations, there are three possible combinations for for x_{11}, x_{21} and x_{31}:

   i. $x_{11} = 1$, $x_{21} = 0$ and $x_{31} = 0$
   ii. $x_{11} = 0$, $x_{21} = 1$ and $x_{31} = 0$
   iii. $x_{11} = 0$, $x_{21} = 0$ and $x_{31} = 1$

   This means $x_{11} + x_{21} + x_{31} = 1$.

   (b) The decision variables are

$$x_{ij} = \begin{cases} 1 & \text{if Database } j \text{ is stored in Location } i \\ 0 & \text{otherwise} \end{cases}$$

We use the notation:

- D_i = capacity of Location i
- t_i = access time of Location i
- s_j = size of Database j
- f_j = frequency of accessing Database j
- F = total number of accesses per second

The (binary) integer programming problem is:

$$\min \frac{1}{F} \sum_{i=1}^{3} \sum_{j=1}^{8} f_j t_i x_{ij}$$

subject to

$$\begin{aligned} \sum_{j=1}^{8} s_j x_{ij} &\leq D_i \quad \text{for } 1 \leq i \leq 3 \\ \sum_{i=1}^{3} x_{ij} &= 1 \quad \text{for } 1 \leq j \leq 8 \\ x_{ij} &\in \{0, 1\} \end{aligned}$$

Note the first set of constraints enforces capacity limit of each location. The second set of constraints ensures that each database is stored at exactly one location.

(c) The location is as showed below.

- Databases 2,3,4 at Location 1.
- Databases 1,7,8 at Location 2.
- The rest at Location 3.

The files used to solve this problem are `hw_db.mod`, `hw_db.dat` and `hw_db_batch`.