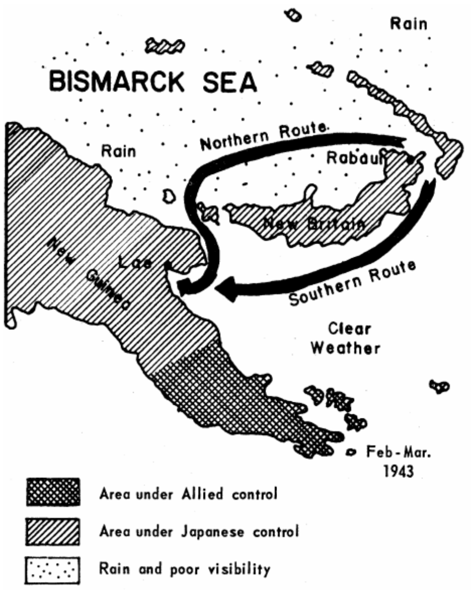
GSOE9210 Engineering Decisions

Problem Set 08

1. Consider the 'Battle of the Bismarck Sea' discussed in lectures.



   (a) Can this be represented as a zero-sum game?
   (b) Represent the game in extensive form (*i.e.*, as a game tree).
   (c) Represent the game in normal (strategic) form (*i.e.*, as a game matrix).
   (d) Simplify the problem by eliminating dominated strategies.
   (e) Which, if any, are the rational 'solutions' to the game?

2. Consider the 'Jailbreak game' from lectures. Suppose that neither the prisoner (P) nor the guard (G) know the other's move.

   (a) Is the game zero-sum?
   (b) Draw the game tree for this game.
   (c) Convert this to extensive form, with the prisoner as the row player.
   (d) Simplify the game using dominance.
   (e) Repeat the above for the game in which the prisoner knows which wing the guard will patrol (*i.e.*, N or S).

3. Use dominance to solve the following zero-sum game (payoffs are for the row player):

	b_1	b_2	b_3	b_4
a_1	0	1	7	7
a_2	4	1	2	10
a_3	3	1	0	25
a_4	0	0	7	10

4. Use dominance to reduce the following zero-sum games:

	b_1	b_2	b_3
a_1	3	8	3
a_2	0	1	10
a_3	3	6	5

	b_1	b_2	b_3	b_4
a_1	1	2	3	3
a_2	1	5	0	0
a_3	1	6	4	1

5. Consider the following zero-sum game, in which mixed strategies are allowed by the players.

		B	
		b_1	b_2
A	a_1	4	0
	a_2	1	4
	a_3	2	1

   (a) Which, if any, strategies can be eliminated by using dominance?
   (b) Show that if player A had a possible strategy a^*, with payoffs 2 and 3 in response to player B's strategies b_1 and b_2 respectively, then a^* would not be dominated.

6. Use dominance to solve the following matrix representation of a two-player non strictly competitive game.

		Y		
		b_1	b_2	b_3
X	a_1	$0,0$	$1,2$	$0,2$
	a_2	$1,3$	$1,4$	$0,0$

7. Two companies, X and Y, produce a similar product which earns a profit of \$1 per unit sold. The two companies compete for a total annual market of 4000 units. However, if either company (or both) advertises, the total annual market will increase by 50%.

   If neither or both companies advertise then they split the market evenly. If only one advertises, then the one that advertises gains two-thirds of the market.

Company X is deciding whether to close production (exit this market), or continue, and if so, whether to advertise or not.

Company Y is committed to this market (*i.e.*, it won't leave), but is monitoring whether ot not company X stays in the market before deciding whether ot not to advertise.

In any case, both companies must decide whether or not to advertise this year before they know whether the other will.

(a) Draw the extensive form of this game.

(b) Draw the corresponding game matrix from the perspective of player X.

(c) Reduce this game to identify possible solutions.

(d) Repeat the above for the case where the annual cost of advertising for each company is $1000.