COMP4418: Knowledge Representation and Reasoning—Exercise Set 2
First-Order Logic

1. Translate the following first-order sentences into English
   (i) ∀x(Bird(x) → Flies(x))
   (ii) ∀x∃y(Person(x) → Mother(y, x))
   (iii) ∃x∀y(Person(x) ∧ Mother(x, y))

Where:
- Bird(x) means “x is a bird”
- Flies(x) means “x flies”
- Person(x) means “x is a person”
- Mother(x, y) means “x is the mother of y”

2. Convert the following English sentences into sentences of first-order logic:
   (i) All cats are mammals.
   (ii) No cat is a reptile.
   (iii) All computer scientists like some operating system.

Use meaningful predicate names or state the scheme of abbreviation that you are using.

3. Convert the following first-order sentences into conjunctive normal form:
   (i) ∀x(Bird(x) → Flies(x))
   (ii) ∃x∀y∀z(Person(x) ∧ ((Likes(x, y) ∧ y ≠ z) → ¬Likes(x, z)))

4. Determine whether the following are valid inferences in first-order logic using resolution.
   (i) ∀x(P(x) → Q(x)) ⊢ ∀y(¬Q(y) → ¬P(y))
   (ii) ∀x(P(x) → Q(x)) ⊢ ∀x(¬Q(x) → ¬P(x))
   (iii) ∀x(P(x) → Q(x)), P(a) ⊢ Q(a)
   (iv) ∀x(P(x) → Q(x)), ∃xP(x) ⊢ ∃xQ(x)
   (v) ∀x(P(x) → Q(x)), ∀x(Q(x) → R(x)) ⊢ ∀x(P(x) → R(x))

5. Consider the following three sentences:
   (A) There is a computer scientist who likes every operating system
   (B) Linux is an operating system
   (C) Someone likes Linux

We wish to investigate the relationship among these three sentences:
   (i) Write a formula in first-order logic expressing each of the given facts. Call them A, B and C.
   (ii) Write the set of clauses corresponding to A, B and ¬C.
   (iii) Derive the empty clause from this set of clauses using resolution.
   (iv) Is there an SLD resolution of the empty clause? Why or why not?
   (v) Explain what entailment relation the resolution derivation shows among the three sentences.