

## COMP4418: Knowledge Representation and Reasoning—Exercise Set 2

### First-Order Logic

1. Translate the following first-order sentences into English

- (i)  $\forall x(Bird(x) \rightarrow Flies(x))$
- (ii)  $\forall x\exists y(Person(x) \rightarrow Mother(y, x))$
- (iii)  $\exists x\forall y(Person(x) \wedge Mother(x, y))$

Where:

- $Bird(x)$  means “x is a bird”
- $Flies(x)$  means “x flies”
- $Person(x)$  means “x is a person”
- $Mother(x, y)$  means “x is the mother of y”

2. Convert the following English sentences into sentences of first-order logic:

- (i) All cats are mammals.
- (ii) No cat is a reptile.
- (iii) All computer scientists like some operating system.

Use meaningful predicate names or state the scheme of abbreviation that you are using.

3. Convert the following first-order sentences into conjunctive normal form:

- (i)  $\forall x(Bird(x) \rightarrow Flies(x))$
- (ii)  $\exists x\forall y\forall z(Person(x) \wedge ((Likes(x, y) \wedge y \neq z) \rightarrow \neg Likes(x, z)))$

4. Determine whether the following are valid inferences in first-order logic using resolution.

- (i)  $\forall x(P(x) \rightarrow Q(x)) \vdash \forall y(\neg Q(y) \rightarrow \neg P(y))$
- (ii)  $\forall x(P(x) \rightarrow Q(x)) \vdash \forall x(\neg Q(x) \rightarrow \neg P(x))$
- (iii)  $\forall x(P(x) \rightarrow Q(x)), P(a) \vdash Q(a)$
- (iv)  $\forall x(P(x) \rightarrow Q(x)), \exists xP(x) \vdash \exists xQ(x)$
- (v)  $\forall x(P(x) \rightarrow Q(x)), \forall x(Q(x) \rightarrow R(x)) \vdash \forall x(P(x) \rightarrow R(x))$

5. Consider the following three sentences:

- (A) There is a computer scientist who likes every operating system
- (B) Linux is an operating system
- (C) Someone likes Linux

We wish to investigate the relationship among these three sentences:

- (i) Write a formula in first-order logic expressing each of the given facts. Call them  $A$ ,  $B$  and  $C$ .
- (ii) Write the set of clauses corresponding to  $A$ ,  $B$  and  $\neg C$ .
- (iii) Derive the empty clause from this set of clauses using resolution.
- (iv) Is there an SLD resolution of the empty clause? Why or why not?
- (v) Explain what entailment relation the resolution derivation shows among the three sentences.