Exercise sheet 4 – Solutions

COMP6741: Parameterized and Exact Computation

Serge Gaspers

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Exercise 1. Recall that an *independent set* of a graph $G = (V, E)$ is a subset of pairwise non-adjacent vertices.

Independent Set	
Input:	Graph G, integer k
Question:	Does G have an independent set of size k?

- Show that Independent Set is FPT for parameter q, where $q = k + \Delta(G)$ and $\Delta(G)$ denotes the maximum degree of G.

Solution sketch. We will restrict our attention to *maximal* independent sets, where we use the following property: if v is a vertex of G, then every maximal independent set contains at least one vertex from $N_G[v]$.

- Select a vertex $v \in V$
- Do a $(d_G(v)+1)$-way branching, recursively checking for each $u \in N_G[v]$, whether $G - N_G[u]$ has an independent set of size at least $k - 1$
- Since k decreases by at least 1 in each branch, and the number of branches is at most $\Delta(G) + 1$, we obtain a running time of $O^*((\Delta(G) + 1)^k)$
- This is an FPT algorithm