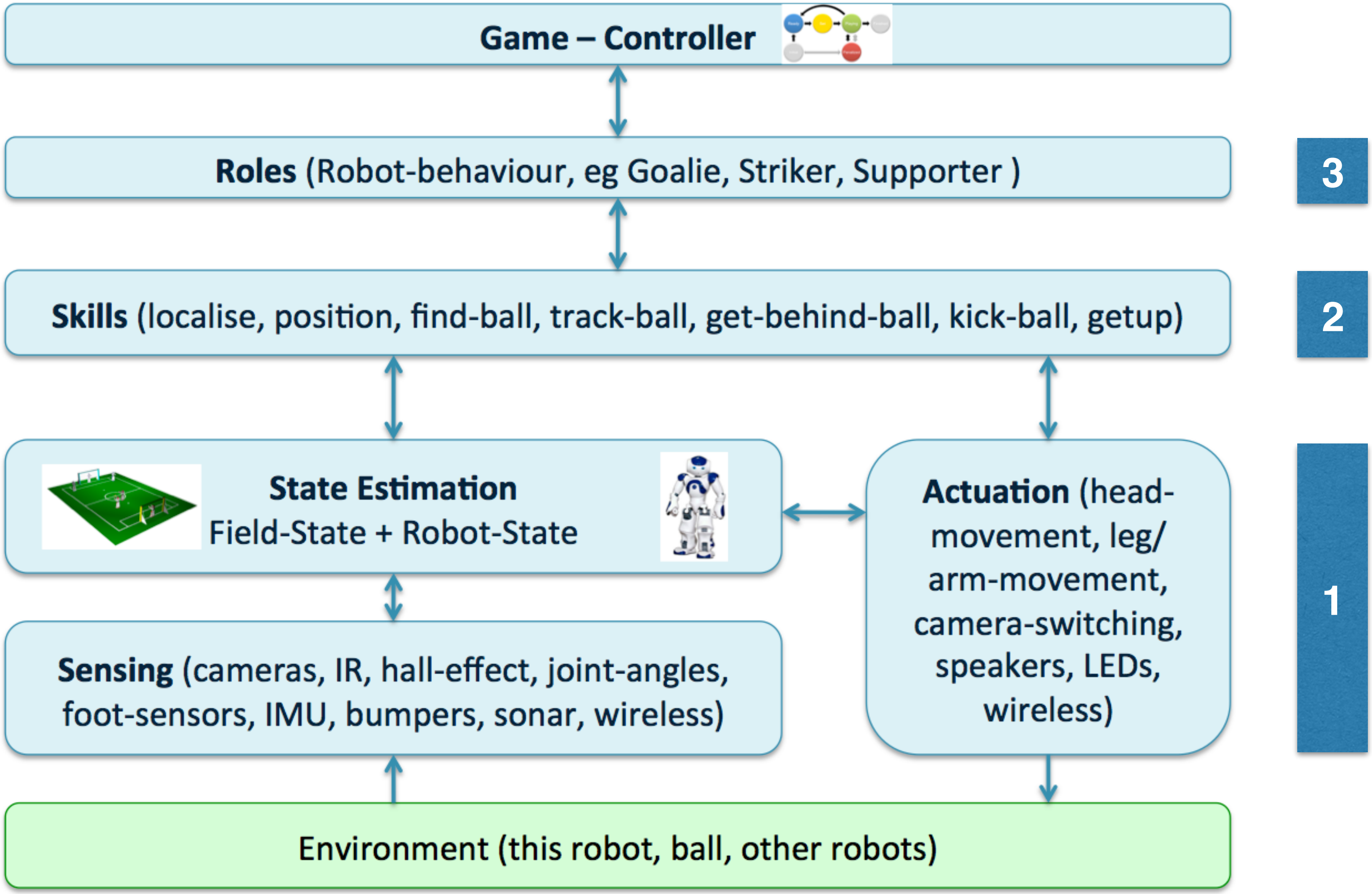
Knowledge Representation and Reasoning

COMP3431 Robot Software Architectures

A Three-Level Architecture

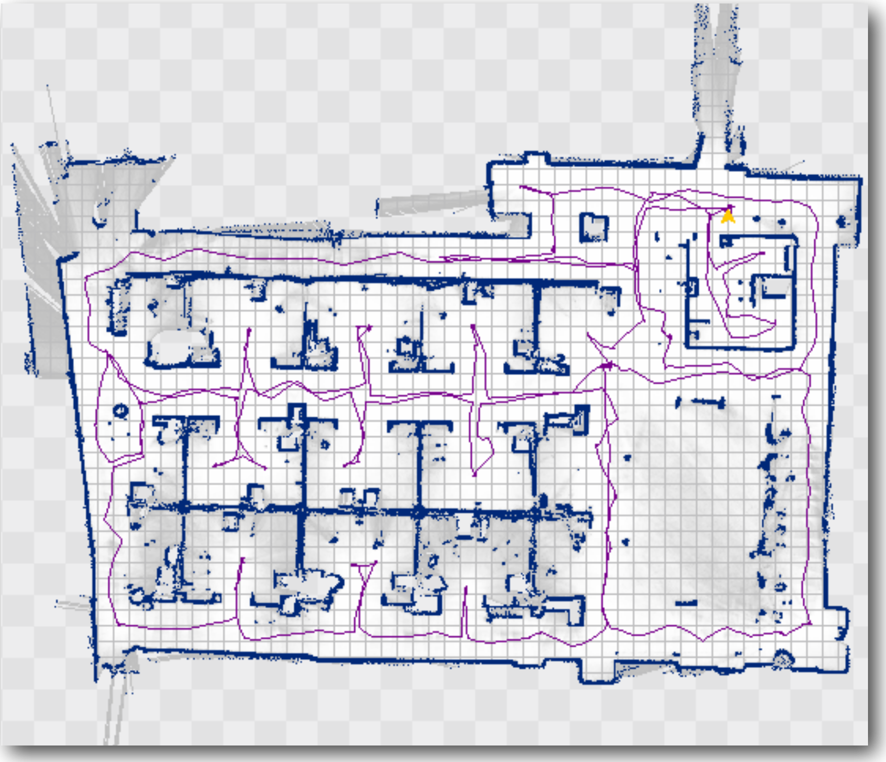


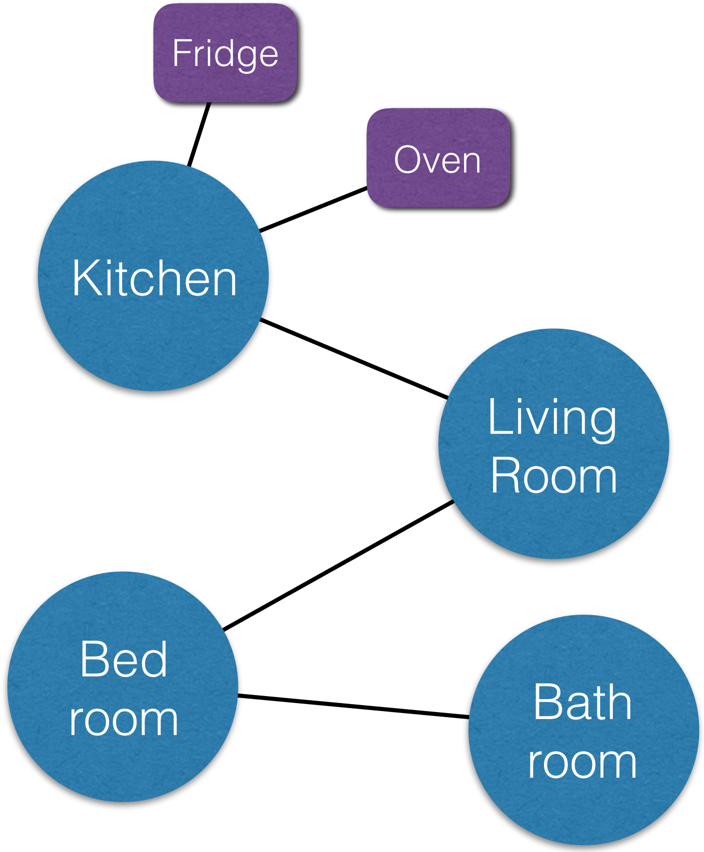
Where are we now?

- We’ve done a whirlwind tour of perception and action
- Now moving up to planing and problem solving
    - and the kind of learning that goes with them

Why do we need symbols?

- How do we ask “where is Tim’s office”?
- How do we know that if we want to get a cold drink, we should find the fridge and it’s probably in the kitchen?





Automated Reasoning

- Expressions in a formal language conform to unambiguous rules of construction.
- Inferences are drawn by following strict laws for manipulating expressions in a formal language
- The language we use most often is clausal form logic.

Propositional Calculus

- A propositional constant is a symbol (like p, q, r, ...) that stands for some like "Sydney is a city”.
- Propositions are atomic formulae.
- A well-formed (wff) formula is
  - an atom, Ψ
  - the negation of a wff, $\neg\Psi$
  - the disjunction (or) of a pair of wffs, $\Psi \vee \Phi$
- Everything else can be derived

Derived Expressions

- $\Psi \land \Phi$ is defined as $\neg(\neg\Psi \lor \neg\Phi)$
- $\Psi \supset \Phi$ is defined as $\neg\Psi \lor \Phi$
- $\Psi \equiv \Phi$ is defined as $(\Psi \supset \Phi) \land (\Phi \supset \Psi)$

Predicate Calculus

- Propositional calculus cannot deal with statements of generality like,

'All men are mortal'

- To do this, we need predicates, arguments, variables and quantifiers. eg.

$$(\forall X)(man(X) \supset mortal(X))$$

Clausal Form

- In clausal form, positive literals are placed to the left of an arrow symbol and negative atoms to the right, e.g.

$$p, q \leftarrow p$$
$$p, q \leftarrow q$$

- In general, a clause is an expression of the form:

$$p_1, \ldots, p_m \leftarrow q_1, \ldots, q_n$$

- The literals on the left are disjoined conclusions.

- The literals on the right are conjoined conditions.

Horn Clauses

- A Horn clause is one which only has a single positive literal, eg.

$$p_1 \leftarrow q_1, \ldots, q_n$$

- The programming language, Prolog, consists of Horn clause definitions, eg.

```
on(a, b).
on(b, c).
above(X, Y) :- on(X, Y).
above(X, Y) :- on(Z, Y), above(X, Z).
```

Resolution

- To prove *p* follows from some theory, *T*, assume ¬*p* and then try to derive a contradiction from its conjunction with *T*.
- Resolution requires a pattern matching operation, called *unification*.
- When matching literals, we look for variable substitutions that will make the two expressions identical. Eg.

  runs_faster_than(X, zeno)

  runs_faster_than(tortoise, Y)

are identical under the substitution {X/tortoise, Y/zeno}

Resolving Clauses

- A clause that contains no variables is called a *ground clause*.
- To resolve two non-ground clauses, you must find a unifier for complimentary literals. Eg.

{beats_in_race(X, zeno), ¬ younger_than(X, zeno)}

and

{¬ beats_in_race(tortoise, Y), ¬ philosopher(Y)}

have unifier n = {X/tortoise, Y/zeno} and generate the resolvent

{¬ philosopher(zeno), ¬ younger_than(tortoise, zeno)}

Proofs

- We can *prove* a formula, *p*, if we can derive it from a theory, *T*, by a sequence of resolution steps.

  Written as $T \vdash p$.

- If the theory is very large, there may be many ways of deriving a proof.
- How can we find a *short* derivation?
- We try a proof by *refutation*, ie. add negation of goal to theory and show that the new theory is inconsistent, ie. implies *false*.
- The empty clause, {}, is interpreted as *false*. So if theory derives *false*, we have an inconsistent theory.

A Prolog Proof Tree

on(a, b).
on(b, c).
above(X, Y) :- on(X, Y).
above(X, Y) :- on(Z, Y), above(X, Z).

```
:- above(a, c).                    above(X, Y) :- on(Z, Y), above(X, Z).

:- on(Z, c), above(a, Z).          on(b, c).

:- above(a, b).                    above(X, Y) :- on(X, Y).

:- on(a, b).                       on(a,  b).

{}
```

Resolution Search

- Resolution uses backward chaining to focus search for clauses to resolve.
- There are many refinements to this search.
- We will stick to the Prolog method which resolves clauses and their literals in input order, ie, top-to-bottom, left-to-right.

Soundness and Completeness

- A proof procedure is sound if every formula it derives is true. I.e. it cannot prove something it shouldn't.
- A proof procedure is complete if it can derive every thing that is possible to derive from a theory. Ie. There is no true statement that it cannot prove.
- Decidability means that we can always show if a proposition follows from a theory.
- Prolog's proof procedure is sound and complete for Horn clauses.
- Unrestricted first-order logic is undecidable.