THE UNIVERSITY OF NEW SOUTH WALES
SEMESTER 2 2016
COMP6741: PARAMETERIZED AND EXACT COMPUTATION
Mid-session Quiz

1. TIME ALLOWED – 1 hour

2. READING TIME – 0 minutes

3. THIS EXAMINATION PAPER HAS 3 PAGES

4. TOTAL NUMBER OF QUESTIONS – 4

5. TOTAL MARKS AVAILABLE – 100

6. ALL QUESTIONS ARE NOT OF EQUAL VALUE. MARKS AVAILABLE FOR EACH QUESTION ARE SHOWN IN THE EXAMINATION PAPER.

7. ALL ANSWERS MUST BE WRITTEN IN INK. EXCEPT WHERE THEY ARE EXPRESSLY REQUIRED, PENCILS MAY BE USED ONLY FOR DRAWING, SKETCHING OR GRAPHICAL WORK.

8. THIS PAPER MAY BE RETAINED BY CANDIDATE.

SPECIAL INSTRUCTIONS

9. ANSWER ALL QUESTIONS.

10. CANDIDATES MAY BRING TO THE EXAMINATION: UNSW approved calculator, all textbooks and lecture notes (handwritten or printed), private documents, etc., but no electronic material and no electronic devices.

11. THE FOLLOWING MATERIALS WILL BE PROVIDED: answer booklet
Your answers may rely on theorems, lemmas and results stated in the lecture notes.

1 Search trees

What is the best running time upper bound you can claim for an algorithm that spends polynomial time at each node of the search tree, has no simplification rules, and has a unique 2-way branching rule that decreases a parameter $k$ by 1 in the first branch and by 2 in the second branch?

Possible options:

- $O^*(2^{k/2}) \subseteq O^*(1.4143^k)$,
- $O^*(\phi^k)$, where $\phi = \frac{1+\sqrt{5}}{2} \approx 1.6181$ is the positive root of $x^{-1} + x^{-2} - 1$,
- $O^*(\rho^k)$, where $\rho = \frac{\sqrt{13}-1}{2} \approx 1.3027$ is the positive root of $x^2 + x - 3$.

2 Self-reducibility

Recall that a leaf of a tree is a vertex with degree 1. A spanning tree in a graph $G = (V,E)$ is a subgraph of $G$ that is a tree and has $|V|$ vertices.

Consider the Maximum Leaf Spanning Tree problem.

<table>
<thead>
<tr>
<th>Maximum Leaf Spanning Tree</th>
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<tbody>
<tr>
<td>Input: connected graph $G$, integer $k$</td>
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<tr>
<td>Parameter: $k$</td>
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<tr>
<td>Question: Does $G$ have a spanning tree with at least $k$ leaves?</td>
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</tbody>
</table>

Suppose $A$ is an algorithm solving Maximum Leaf Spanning Tree in $O^*(3.72^k)$ time.

- Design an algorithm $B$, which, for an input graph $G$, outputs a spanning tree with a largest number of leaves of $G$ in $O^*(3.72^k)$ time, where $k^*$ is the largest number of leaves in any spanning tree of $G$.

Note: Feel free to use polynomial-time algorithms by Prim or Kruskal for computing an arbitrary spanning tree of a graph or a spanning tree with maximum weight (when edges are weighted) without describing them.

3 Max Cut

A cut in a graph $G = (V,E)$ is a partition of the vertex set $V$ into two sets $U$ and $W$. The size of a cut is the number of edges with one endpoint in $U$ and the other endpoint in $W$, i.e., $|\{uv \in E : u \in U \text{ and } v \in W\}|$. Consider the Max Cut problem.

<table>
<thead>
<tr>
<th>Max Cut</th>
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<tbody>
<tr>
<td>Input: A graph $G = (V,E)$, an integer $k$</td>
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<tr>
<td>Parameter: $k$</td>
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<tr>
<td>Question: Does $G$ have a cut of size at least $k$?</td>
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</tbody>
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1. Design a simplification rule that removes isolated vertices. [10 marks]

2. Design a simplification rule that removes vertices of degree 1. [10 marks]

3. Obtain a kernel with $O(k)$ vertices and edges based on the following fact: there exists a labeling of the vertices with 0’s and 1’s (one label per vertex) where at least $|E|/2$ edges have distinct labels on their endpoints.

   **Note:** The fact can be proved by a simple probabilistic argument: If we randomly label the vertices of $G$ with 0 and 1, the expected number of edges where the endpoints have distinct labels is $|E|/2$. Therefore, there exists at least one such labeling where at least $|E|/2$ edges have distinct labels on their endpoints. [20 marks]

4 **Hitting Set** [30 marks]

A *set system* is a pair $(V, H)$ where $V$ is a finite ground set of elements, and $H$ is a collection of subsets of $V$. The *rank* $r(S)$ of a set system $S = (V, H)$ is the size of the largest set in $H$, i.e., $r(S) = \max_{X \in H} |X|$. A *hitting set* of a set system $(V, H)$ is a subset of elements $S \subseteq V$ such that each set in $H$ contains at least one element from $S$.

<table>
<thead>
<tr>
<th><strong>Sol+Rank Hitting Set</strong></th>
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<td><strong>Input:</strong></td>
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1. Draw or describe a No-instance with 5 elements, rank 3, and $k = 2$. [5 marks]

2. Design an algorithm solving **Sol+Rank Hitting Set** in $O^*((r(S))^k)$ time. [20 marks]

3. Is this an FPT algorithm? [5 marks]