Solution to COMP9334 Revision Questions for Week 2 — Part 2

Question on Poisson Process

In order to refer to the two Poisson processes in a convenient way, I call them P_1 and P_2. The Poisson processes P_1 and P_2, have rates r_1 and r_2, respectively.

Consider a time interval T. Since P_1 is a Poisson process with rate r_1, we know that the probability that there are k arrivals in time interval T is

$$\frac{e^{-r_1 T}(r_1 T)^k}{k!} \tag{1}$$

Similarly, the probability that there are j arrivals in time interval T from P_2 is

$$\frac{e^{-r_2 T}(r_2 T)^j}{j!} \tag{2}$$

Let us consider the aggregation of the two Poisson processes P_1 and P_2 over the time interval T. The arrivals can come from P_1 or P_2. Let us find the probability that there are n arrivals in T. If there are n arrivals from P_1 and P_2 together, this can be resulted from

- 0 arrivals from P_1 and n arrivals from P_2
- 1 arrivals from P_1 and $(n-1)$ arrivals from P_2
- 2 arrivals from P_1 and $(n-2)$ arrivals from P_2
  ...
- $(n-1)$ arrivals from P_1 and 1 arrivals from P_2
- n arrivals from P_1 and 0 arrivals from P_2

Therefore

$$\begin{aligned}
& \text{Probability that there are } n \text{ arrivals over time } T \text{ from } P_1 \text{ and } P_2 \text{ together} \\
= & \sum_{i=0}^{n} \text{Probability of } i \text{ arrivals over time } T \text{ from } P_1 \times \text{Probability of } (n-i) \text{ arrivals over time } T \text{ from } P_2 \\
= & \sum_{i=0}^{n} \frac{e^{-r_1 T}(r_1 T)^i}{i!} \frac{e^{-r_2 T}(r_2 T)^{n-i}}{(n-i)!} \\
= & \frac{1}{n!} e^{-(r_1+r_2)T} \sum_{i=0}^{n} \frac{n!}{i!(n-i)!} (r_1 T)^i (r_2 T)^{(n-i)} \\
= & \frac{1}{n!} e^{-(r_1+r_2)T} ((r_1+r_2)T)^n
\end{aligned}$$

This shows that the aggregation of P_1 and P_2 is a Poisson process with rate $r_1 + r_2$.