# GSOE9210 Engineering Decisions

## Problem Set 04

1. At a school fête, a suspicious-looking man is offering bets on the toss of a coin of questionable fairness. The man is offering \$2 for each dollar bet (plus your original dollar) if the contestant chooses the face on the coin correctly.

   (a) Suppose Alice has a \$10 note in her pocket; represent the gamble as a decision table. Include the option of leaving (L) (*i.e.*, refusing to gamble).

   (b) If Alice were pessimistic (*i.e.*, she used *Maximin* as her decision rule), would Alice bet on heads or tails, or not bet?

   (c) Suppose Alice has a similarly pessimistic friend Bob, and both could bet together on the same toss, would that affect Alice's decision?

   (d) Suppose Alice was friendless but had—instead of one \$10 note—\$10 \$1 coins in her pocket. How might this affect her decision?

2. (a) Find the value $\frac{3}{4}$ of the way from 4 to 2 on the number line.

   (b) Find a general expression for the value $\mu$ of the way from $a$ to $b$ on the number line.

   Consider the decision problem below:

   | | $s_1$ | $s_2$ |
   |---|---|---|
   | A | 2 | 3 |
   | B | 4 | 0 |
   | C | 3 | 3 |
   | D | 5 | 2 |
   | E | 3 | 5 |

   (c) Plot the actions as points on the Cartesian plane. Find the coordinates of the point $\mu$ of the way from A to B.

   (d) Find the point, $P(x, y)$, on the segment AB that intersects with the diagonal $y = x$.

   (e) Find the *Maximin* mixed action for the decision problem above.

3. Consider a scenario with four states, two pure actions, and their mixtures ($M$), where $\mu_A = \mu$:

   | | $s_1$ | $s_2$ | $s_3$ | $s_4$ |
   |---|---|---|---|---|
   | A | 4 | 2 | 1 | 3 |
   | B | 0 | 1 | 5 | 2 |

(a) Draw the mixture plot for this problem.
(b) Do all states need to be considered when determining the *Maximin* mixed action?
(c) Find the *Maximin* mixed action, and the *Maximin* value—*i.e.*, the value of the *Maximin* mixed action—for this problem.

4. A drinks seller can purchase stock of several types of drink: a) hot chocolate; b) iced tea; c) lemonade; d) orange juice.

   She knows, from past experience, that on warm days she'll make sales totalling \$10 on hot chocolate, \$40 on iced tea, \$30 on lemonade, and \$40 on orange juice. On cool days, however, her sales total \$30 on hot chocolate, \$0 on iced tea, \$20 on lemonade, and \$10 on orange juice.

   (a) Produce a decision table for this problem.
   (b) What proportion of drinks should she stock to maximise her guaranteed sales total regardless of the temperature?
   (c) Draw the admissibility frontier for this problem. Are any actions inadmissible?

5. Alice is considering whether to invest \$1000 over an investment period, and if so on which option to invest. She is looking at an investment market which will either rise ($r$) by 6% or flatten ($f$) to 0% over the investment period. She will learn the market movement mid-period, at which point she can choose to invest in the fixed rate option (F), which gives a constant return of 2%, or risky option take a (R) which follows the market's movement.

   For the fixed option, Alice has the added option to invest initially for a long term (L) (*i.e.*, for the full period), or a short term (S) (half period). If she invests short term she will respond to the market in either of two ways: (a) if the market has risen she'll change to the risky option, earning an average profit of 4%; (b) if the market has fallen she'll reinvest in F but will receive a lower rate, which would reduce her gains to 1% per annum.

   She must submit her investment instructions ($\bar{\text{I}}$, R, or F, and for the latter, L or S) to her stock broker before the market's movement is known.

   (a) Represent this situation as a decision tree (*i.e.*, in extensive form) and as a decision table.
   (b) How many information sets are there in this problem?
   (c) Assuming diversified stock portfolios (*i.e.*, mixtures of investments) are allowed, which mixed strategies are admissible?
   (d) Which is the *Maximin* mixed strategy?
   (e) Which is the *miniMax Regret* mixed strategy?

6. Consider the following decision table, in which mixed strategies are allowed.

| | $s_1$ | $s_2$ |
|---|---|---|
| A | 4 | 0 |
| B | 1 | 4 |
| C | 2 | 1 |

(a) Which, if any, strategies are dominated?

(b) Prove that a possible strategy S, with payoffs 2 and 3 in states $s_1$ and $s_1$ respectively, would not be dominated.