Assignment 1
COMP6741: Parameterized and Exact Computation

2015, Semester 2

Assignment 1 is based on group work. Each group consists of 2–3 students. Each group selects a name for the group and one corresponding author, who will be responsible for the submission of the assignment. Register your group on [WebCMS3](#) with type “Assignment1”. Post on the Forum if you are looking for a group to join, or if your group is short of members.

For the solutions to this assignment, you may rely on all theorems, lemmas, and results from the lecture notes. If any other works (articles, Wikipedia entries, lecture notes from other courses, etc.) inspired your solutions, please cite them and give a list of references at the end.

If you have questions about this assignment, please post them to the Forum.

**Due date.** This assignment is due on Tuesday, 25 August 2015, at 23.59 AEST. Submitting $x$ days after the deadline, with $x > 0$, reduces the grade by $20 \cdot x$ per cent.

**How to submit.** Submit a PDF with your solutions using the command

```
give cs6741 a1 <mysolution.pdf>
```

from the CSE network, or use the new [WebCMS3 frontend](#) for `give`.

1. In the **Meeting Most Deadlines** problem, we are given $n$ tasks $t_1, \ldots, t_n$, and each task $t_i$ has a length $\ell_i$, a due date $d_i$, and a penalty $p_i$ which applies when the due date of task $t_i$ is not met. The problem asks to assign a start date $s_i \geq 0$ to each task $t_i$ so that the executions of no two tasks overlap, and the sum of the penalties of those tasks that are not finished by the due date is minimized.

**Meeting Most Deadlines**

| Input:   | A set $T = \{t_1, \ldots, t_n\}$ of $n$ tasks, where each task $t_i$ is a triple $(\ell_i, d_i, p_i)$ of three non-negative integers. |
| Output:  | A schedule, assigning a start date $s_i \in \mathbb{N}_0$ to each task $t_i \in T$ such that $\sum_{i \in \{1, \ldots, n\} : s_i + \ell_i > d_i} p_i$ is minimized, subject to the constraint that for every $i, j \in \{1, \ldots, n\}$ with $i \neq j$ we have that $s_i \notin \{s_j, s_j + 1, \ldots, s_j + \ell_j - 1\}$. |

- Show that the **Meeting Most Deadlines** problem can be solved in $O^*(n!)$ time by reformulating it as a permutation problem. \[10 \text{ points}\]
- Design an algorithm solving the **Meeting Most Deadlines** problem in $O^*(2^n)$ time. \[25 \text{ points}\]

[At the cost of $5$ points, you may assume that all integers in the problem formulation have value at most $n^{O(1)}$.]
2. Consider the 3-IN-5-Sat problem, where all clauses have length 5 and we need to satisfy exactly 3 literals of every clause.

<table>
<thead>
<tr>
<th>3-IN-5-Sat</th>
</tr>
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<tbody>
<tr>
<td><strong>Input:</strong></td>
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<td><strong>Question:</strong></td>
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Example:

\[
F = (a \lor \neg b \lor \neg c \lor \neg d \lor e) \land (\neg a \lor c \lor \neg d \lor \neg e \lor f) \land (a \lor b \lor d \lor e \lor \neg f)
\]

is a Yes-instance for 3-IN-5-Sat, since the assignment \( \alpha \) with \( \alpha(a) = \alpha(b) = \alpha(c) = \alpha(e) = \alpha(f) = 1 \) and \( \alpha(d) = 0 \) satisfies 3 literals in each clause of \( F \).

- Design and analyze an algorithm for 3-IN-5-Sat. \[35\text{ points}\]
  [The number of awarded points for a correct algorithm and analysis depends on the running time: \( O^*(2^n) \) gives 5 points, \( O^*(1.8^n) \) gives 10 points, \( O^*(1.62^n) \) gives 20 points, \( O^*(1.59^n) \) gives 30 points, and \( O^*(1.58^n) \) gives 35 points, where \( n \) is the number of variables of \( F \).]

3. A **domatic k-partition** of a graph \( G = (V, E) \) is a partition \( (D_1, \ldots, D_k) \) of \( V \) into \( k \) dominating sets of \( G \). The **domatic number** of \( G \) is the largest integer \( k \) such that \( G \) has a domatic \( k \)-partition.

- Design a \( O^*(2^n) \) time inclusion-exclusion algorithm that computes the domatic number of any graph on \( n \) vertices. \[15\text{ points}\]
- Design and analyze a polynomial-space inclusion-exclusion algorithm that computes the domatic number of any graph. \[15\text{ points}\]