COMP2111 Week 5 Term 1, 2019 Predicate Logic: Tools

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Logic, computationally

- Easy problems: verifying a proof, checking a satisfying model
- Hard problems: finding a proof, finding a satisfying model



- Theorem provers
- SMT solvers
- Knowledge Based Systems



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Theorem provers

Proof assistants:

- Interactive/directed theorem proving
- Some have automated theorem proving
- Minimal default behaviour
 - Can implement a wide range of logical systems
 - Can prove correctness from foundations
- Some can extract code from specifications (e.g. Coq)

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Theorem provers: Examples

- Coq (used to prove 4-colour theorem)
- Isabelle (proved functional correctness of seL4 microkernel)
- HOL (proved Kepler conjecture)
- Natural deduction prover



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Satisfaction Modulo Theories

- SAT solvers for (fragments of) Predicate Logic
- Theories force certain interpretations (e.g. arithmetic)
- Two main approaches:
 - Convert everything into SAT
 - Combination of SAT solver and Theory-specific solvers

Advantages

- Tend to be very fast
- Good at handling domain-specific logic (e.g. arithmetic)

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Disadvantages

- Restricted to quantifier-free fragments
- Unwieldy statements

SMT solvers: Examples

• Z3

- Wide range of built-in theories
- Backend for verification tools such as Dafny (see SENG2011)

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- OpenSMT
- CVC4

Example usage

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Logic puzzles (e.g. Zebra puzzle)

Example

- The Spaniard lives to the right of the red house
- The Norwegian lives in the blue house
- The Italian lives in house #2

Question: Who lives in the white house?

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- Theorem provers
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Knowledge Based Systems

Build up "knowledge" from base facts and inference rules **Advantages**

- Works with unrestricted Predicate logic
- Simpler statements

Disadvantages

- More fine-tuning of constraints required
- Can be quite slow



IDP: Knowledge Based System for Predicate Logic

Example usage

(日) (同) (目) (日) (日) (日)

Logic puzzles (e.g. Zebra puzzle)

Example

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