COMP9334 Tutorial for Week 12 - Solution

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1. Following the hint given in the question, we define the following decision variables:

   - $$x_{ijuv} = \begin{cases} 1 & \text{if the traffic from Node } u \text{ to Node } v \text{ uses link } (i,j) \\ 0 & \text{otherwise} \end{cases}$$

   - $$y_{ijk} = \begin{cases} 1 & \text{if a link of type } k \text{ is to be built from node } i \text{ to node } j \\ 0 & \text{otherwise} \end{cases}$$

   In addition, we define the following:

   - $N = \{1, 2, 3, 4\}$ (i.e. the set of nodes)
   - $E = \{(i, j) \in N \times N : i \neq j\}$ (i.e. the set of all possible links)
   - c_k = the cost of link of type k (as given in Table 2)
   - b_k = the capacity of link of type k (as given in Table 2)
   - f_{uv} = the traffic demand from city u to city v (as given in Table 1)
   - An indicator function

   $$\delta_{pq} = \begin{cases} 1 & \text{if } p = q \\ 0 & \text{otherwise} \end{cases}$$

   The optimisation problem is:

   $$\min \sum_{(i,j)\in E} \sum_{k=1}^{3} y_{ijk} c_k$$

   subject to

   $$\sum_{j:(i,j)\in E} x_{ijuv} - \sum_{j:(j,i)\in E} x_{jiuv} = \delta_{iu} - \delta_{iv} \ \ \forall i \in N, (u,v) \in E \qquad (1)$$

   $$\sum_{(u,v)\in E} x_{ijuv} f_{uv} \leq \sum_{k=1}^{3} b_k y_{ijk} \ \ \forall (i,j) \in E \qquad (2)$$

   $$\sum_{k=1}^{3} y_{ijk} \leq 1 \ \ \forall (i,j) \in E \qquad (3)$$

   $$x_{ijuv} \in \{0,1\} \qquad (4)$$

   $$y_{ijk} \in \{0,1\} \qquad (5)$$

Note:

- Equation 1 is equivalent to

$$\sum_{j:(i,j)\in E} x_{ijuv} - \sum_{j:(j,i)\in E} x_{jiuv} = \begin{cases} 1 & \text{if } i = u \\ 0 & \text{if } i \neq u, v \\ -1 & \text{if } i = v \end{cases}$$

  The expression $\delta_{iu} - \delta_{iv}$ evaluates to the expression on the right-end-side of the above equation.

- Equation 2 ensures: (1) There is enough capacity in the link if it is built; and (2) If a link is not built (i.e. all $y_{ijk} = 0$ for $k = 1, 2, 3$), no traffic is routed in (i, j) (i.e. $x_{ijuv} = 0$).

- Equation 3 ensures that only one type of link is chosen.

The AMPL code is given in `carrier.mod`, `carrier.dat` and `carrier_batch`.

The optimised network has a cost of 58 units. It has 4 links of type 1 and they are (1,4), (2,3), (3,1), (3,2). It also has a link of type 2 for link (4,3). No links of type 3 is used. Traffic from node 1 to node 2 follows the path (1,4),(4,3),(3,2). Traffic from node 2 to node 3 follows the path (2,3). These can be read from x_{ijuv}. See the file `carrier-solution` where you can read out the paths for the other traffic demands.

Note that there are multiple networks (with different links and routes) which also gives the minimum cost of 58 units. These alternatives are also acceptable.

2. The decision variables are

   - $$y_i = \begin{cases} 1 & \text{if a controller is to be placed at node } i \\ 0 & \text{otherwise} \end{cases}$$

   - $$x_{ij} = \begin{cases} 1 & \text{if the controller at node } i \text{ is to control the switch at node } j \\ 0 & \text{otherwise} \end{cases}$$

   The optimisation problem is:

$$\min \sum_{i=1}^{n} y_i$$

   subject to

$$\sum_{i=1}^{n} x_{ij} = 1 \quad \forall j = 1, .., n \tag{6}$$

$$x_{ij} d_{ij} \leq D y_i \quad \forall i, j = 1, .., n \tag{7}$$

$$x_{ij} \in \{0, 1\} \tag{8}$$

$$y_i \in \{0, 1\} \tag{9}$$

Equation 6 enforces the constraint that a switch is associated with only one controller. Equation 7 ensures that: (i) If $y_i = 0$, i.e. there is no controller at node i, then $x_{ij} = 0$; (ii) If $y_i = 1$, then any switch that is connected to the controller at node i must have a communication delay of less than D. Equation 7 has combined (i) and (ii) into a set of inequalities. Alternatively, you can use

$$\begin{aligned} x_{ij} &\leq y_i \;\; \forall i, j = 1, .., n && (10) \\ x_{ij} d_{ij} &\leq D \;\; \forall i, j = 1, .., n && (11) \end{aligned}$$

to separately enforce the conditions (i) and (ii).