1. Consider the river problem described in lectures:

<table>
<thead>
<tr>
<th></th>
<th>p</th>
<th>1 - p</th>
<th>V_B</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>4</td>
<td>0</td>
<td>4p</td>
</tr>
<tr>
<td>B</td>
<td>3</td>
<td>1</td>
<td>2p + 1</td>
</tr>
</tbody>
</table>

(a) For \( p = \frac{3}{4} \), what is the slope of the Bayes indifference line through A?
(b) Draw the Bayes indifference curves for \( p = \frac{1}{4} \) and \( \frac{3}{4} \) through A and B.
(c) Draw the Bayes indifference curve for which an agent would be indifferent between A and B, respectively. What is the slope of the line?
(d) For which probability (i.e., value of \( p \)) would an agent be indifferent between A and B under the Bayes decision rule?
(e) What is the Bayes value associated with the indifference curve through A and B?
(f) For which values of \( p \) would an agent prefer A to B?

2. Repeat the above exercises for regret. What can you infer about the Bayes decision rule when applied to the original values versus regrets?

3. Consider the generic two-strategy problem below:

<table>
<thead>
<tr>
<th></th>
<th>p</th>
<th>1 - p</th>
<th>s_1</th>
<th>s_2</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>a_1</td>
<td>a_2</td>
<td></td>
<td></td>
</tr>
<tr>
<td>B</td>
<td>b_1</td>
<td>b_2</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Assume neither strategy dominates the other.

(a) Prove that an agent will be indifferent between A and B under Bayes when:

\[
p = \frac{\Delta y}{\Delta x + \Delta y}
\]

where

\[
\Delta y = |a_2 - b_2| \\
\Delta x = |a_1 - b_1|
\]
(b) Prove that:
\[ p = \frac{m}{m - 1} \]
where \( m = -\frac{\Delta y}{\Delta x} \) is the slope of the line joining A and B in the Cartesian plane.

4. Consider the decision table below, with \( P(s_1) = p \):

\[
\begin{array}{c|cc}
\text{ } & p & 1 - p \\
\hline
s_1 & A & 5 & 3 \\
  & B & 4 & 1 \\
  & C & 2 & 5 \\
\end{array}
\]

(a) For which value of \( p \) would the agent be indifferent between A and C?
(b) Plot the Bayes values for the strategies as \( p \) varies from 0 to 1.
(c) For which values of \( p \) are A, B, and C preferred, respectively, under the Bayes decision rule?

5. Each day, a drinks vendor must purchase stock of several types of drinks to sell in her shop. She can choose from four types of drink: a) hot chocolate; b) iced tea; c) lemonade; d) orange juice.
She knows, from past experience, that on warm (\( w \)) days she’ll make sales totalling $10 on hot chocolate, $40 on iced tea, $30 on lemonade, and $40 on orange juice. On cool (\( c \)) days, however, her sales total is $30 on hot chocolate, $0 on iced tea, $20 on lemonade, and $10 on orange juice.
Assume days are either warm or cool, but she will not know which before she must order her stock.

(a) Produce a decision table for this problem.
(b) What proportion of drinks should she stock to maximise her guaranteed (i.e., minimum) sales total regardless of the temperature?
(c) Find the Bayes strategies for \( p = 0, \frac{1}{4}, \frac{1}{2}, \frac{3}{4}, 1 \).
(d) What is the least favourable probability distribution on warm and cool (not warm) days?
(e) Repeat the above analysis for the miniMax Regret rule.
(f) Define the admissibility frontier for this problem.

6. Show that a strategy is admissible iff it is a Bayes strategy for some probability distribution.

7. Show that a Maximin strategy is always a Bayes strategy for some probability distribution.

8. Prove that for any two actions A and B, if A weakly dominates B, and all state probabilities are non-zero, then the Bayes decision rule will strictly prefer A over B.