13. Review

COMP6741: Parameterized and Exact Computation

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Outline

1. Review
   - Upper Bounds
   - Lower Bounds

2. Research in Parameterized and Exact Computation

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Dynamic Programming across Subsets

- very general technique
- uses solutions of subproblems
- typically stored in a table of exponential size

Analysis of Branching Algorithm

Lemma 1 (Measure Analysis Lemma)

Let

- A *be a branching algorithm*
- $c \geq 0$ *be a constant, and*
- $\mu(\cdot), \eta(\cdot)$ *be two measures for the instances of* A,

such that on input I, A *calls itself recursively on instances* $I_1, \ldots, I_k$, *but, besides the recursive calls, uses time* $O(|I|^c)$, *such that*

$$(\forall i) \quad \eta(I_i) \leq \eta(I) - 1, \text{ and} \tag{1}$$

$$2^{\mu(I_1)} + \ldots + 2^{\mu(I_k)} \leq 2^{\mu(I)}. \tag{2}$$

Then A *solves any instance* I *in time* $O(\eta(I)^{c+1}) \cdot 2^{\mu(I)}$.

Inclusion-Exclusion

Theorem 2 (IE-theorem – intersection version)

Let $U = A_0$ *be a finite set, and let* $A_1, \dots, A_k \subseteq U$*.*

$$\left| \bigcap_{i \in \{1,\dots,k\}} A_i \right| = \sum_{J \subseteq \{1,\dots,k\}} (-1)^{|J|} \left| \bigcap_{i \in J} \overline{A_i} \right|,$$

where $\overline{A_i} = U \setminus A_i$ *and* $\bigcap_{i \in \emptyset} = U$*.*

Theorem 3

The number of covers with k *sets and the number of ordered partitions with* k *sets of a set system* (V, H) *can be computed in polynomial space and*

1. $O^*(2^n |H|)$ *time if* H *can be enumerated in* $O^*(|H|)$ *time and poly space,*
2. $O^*(3^n)$ *time if membership in* H *can be decided in polynomial time, and*
3. $\sum_{j=0}^{n} \binom{n}{j} T_H(j)$ *time if there is a* $T_H(j)$ *time poly space algorithm to count for any* $W \subseteq V$ *with* $|W| = j$ *the number of sets* $S \in H$ *st.* $S \cap W = \emptyset$*.*

Main Complexity Classes

P: class of problems that can be solved in time $n^{O(1)}$
FPT: class of problems that can be solved in time $f(k) \cdot n^{O(1)}$
W[$\cdot$]: parameterized intractability classes
XP: class of problems that can be solved in time $f(k) \cdot n^{g(k)}$

$$\mathsf{P} \subseteq \mathsf{FPT} \subseteq \mathsf{W}[1] \subseteq \mathsf{W}[2] \cdots \subseteq \mathsf{W}[P] \subseteq \mathsf{XP}$$

Known: If FPT $=$ W[1], then the Exponential Time Hypothesis fails, i.e. 3-SAT can be solved in time $2^{o(n)}$.

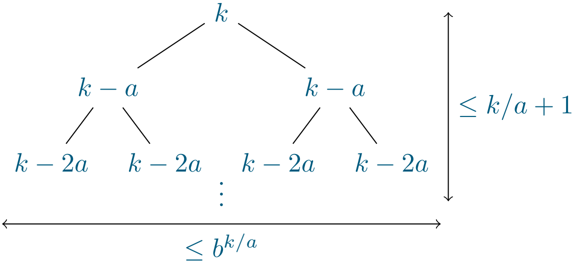
Kernelization: definition

Definition 4

A kernelization for a parameterized problem Π is a **polynomial time** algorithm, which, for any instance I of Π with parameter k, produces an **equivalent** instance I' of Π with parameter k' such that $|I'| \leq f(k)$ and $k' \leq f(k)$ for a computable function f.
We refer to the function f as the size of the kernel.

Search trees

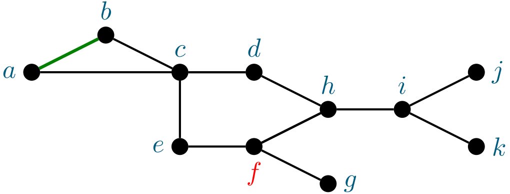
Recall: A search tree models the recursive calls of an algorithm.
For a b-way branching where the parameter k decreases by a at each recursive call, the number of nodes is at most $b^{k/a} \cdot (k/a+1)$.



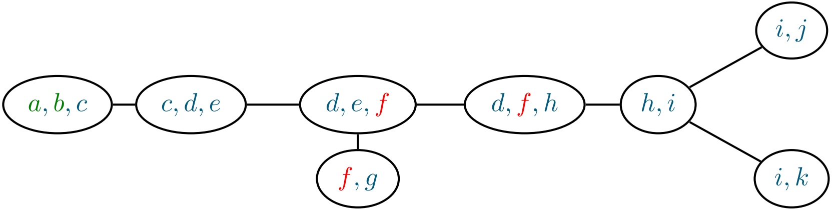
If k/a and b are upper bounded by a function of k, and the time spent at each node is FPT (typically, polynomial), then we get an FPT running time.

Tree decompositions (by example)

- A graph G



- A *tree decomposition* of G



Conditions: covering and connectedness.

Randomized algorithms

Solution intersects a linear number of edges:

- Sampling vertices with probability proportional to their degree gives good success probability if the set of vertices we try to find has large intersection with the edges of the graph.

Color Coding:

Lemma 5

Let $X \subseteq U$ be a subset of size k of a ground set U.
Let $\chi : U \to \{1, \ldots, k\}$ be a random coloring of U.
The probability that the elements of X are colored with pairwise distinct colors is $\geq e^k$.

Monotone Local Search:

- For many subset problems a $O^*(c^k)$ algorithm for finding a solution of size k can be turned into a randomized algorithm finding an optimal solution in time $O^*((2 - 1/c)^n)$.

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Reductions

We have seen several reductions, which, for an instance (I, k) of a problem Π, produce an equivalent instance I' of a problem Π'.

	time	parameter	special features	used for
kernelization	poly	$k' \leq g(k)$	$\lvert I'\rvert \leq g(k)$ $\Pi = \Pi'$	$g(k)$-kernels
parameterized reduction	FPT	$k' \leq g(k)$		W[·]-hardness
OR-composition	poly	$k' \leq \text{poly}(k)$	$\Pi = \text{OR}(\Pi')$	Kernel LBs
AND-composition	poly	$k' \leq \text{poly}(k)$	$\Pi = \text{AND}(\Pi')$	Kernel LBs
polynomial parameter transformation	poly	$k' \leq \text{poly}(k)$		Kernel LBs (S)ETH LBs
SubExponential Reduction Family	subexp(k)	$k' \in O(k)$	Turing reduction $\lvert I'\rvert = \lvert I\rvert^{O(1)}$	ETH LBs

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News

- Recently solved open problems from [Downey Fellows, 2013]
  - BICLIQUE is W[1]-hard [Lin, SODA 2015]
  - SHORT GENERALIZED HEX is W[1]-complete [Bonnet, Gaspers, Lambillotte, Rümmele, Saffidine, ICALP 2017]
  - Determining the winner of a PARITY GAME is FPT in the number of values [Calude, Jain, Khoussainov, Li, Stephan, STOC 2017]
- research focii
  - enumeration algorithms and combinatorial bounds
  - randomized algorithms
  - backdoors
  - treewidth: computation, bounds on the treewidth of grid or planar subgraphs / minors
  - bidimensionality
  - bottom-up: improving the quality of subroutines of heuristics
  - (S)ETH widely used now, also for poly-time lower bounds
  - quests for multivariate algorithms, lower bounds for Turing kernels
  - FPT-approximation algorithms, lossy kernels

Resources

- FPT wiki: http://fpt.wikidot.com
- FPT newsletter: http://fpt.wikidot.com/fpt-news:the-parameterized-complexity-newsletter
- cstheory stackexchange: http://cstheory.stackexchange.com
- FPT summer schools (include lecture slides)
  - 2017: https://algo2017.ac.tuwien.ac.at/pcss/
  - 2014: http://fptschool.mimuw.edu.pl
  - 2009: http://www-sop.inria.fr/mascotte/seminaires/AGAPE/

Advertisements

- Data61 6th Optimisation Summer School, 14-19 Jan 2018 in Kioloa, NSW
- UNSW Algorithms group

Please fill in the myExperience course survey if you have not done so yet.