1. Consider the river problem described in lectures:

<table>
<thead>
<tr>
<th>p</th>
<th>1 - p</th>
<th>V_B</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>4</td>
<td>0</td>
</tr>
<tr>
<td>B</td>
<td>3</td>
<td>1</td>
</tr>
</tbody>
</table>

(a) For \( p = \frac{3}{4} \), what is the slope of the Bayes indifference line through A?

(b) Draw the Bayes indifference curves for \( p = \frac{1}{4} \) and \( \frac{3}{4} \) through A and B.

(c) Draw the Bayes indifference curve for which an agent would be indifferent between A and B, respectively. What is the slope of the line?

(d) For which probability (i.e., value of \( p \)) would an agent be indifferent between A and B under the Bayes decision rule?

(e) What is the Bayes value associated with the indifference curve through A and B?

(f) For which values of \( p \) would an agent prefer A to B?

2. Repeat the above exercises for regret. What can you infer about the Bayes decision rule when applied to the original values versus regrets?

3. Consider the generic two-strategy problem below:

<table>
<thead>
<tr>
<th>p</th>
<th>1 - p</th>
<th>s_1</th>
<th>s_2</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>a_1</td>
<td>a_2</td>
<td></td>
</tr>
<tr>
<td>B</td>
<td>b_1</td>
<td>b_2</td>
<td></td>
</tr>
</tbody>
</table>

Assume neither strategy dominates the other.

(a) Prove that an agent will be indifferent between A and B under Bayes when:

\[
p = \frac{\Delta y}{\Delta x + \Delta y}
\]

where

\[
\Delta y = |a_2 - b_2|
\]
\[
\Delta x = |a_1 - b_1|
\]
(b) Prove that:
\[ p = \frac{m}{m - 1} \]

where \( m = \frac{\Delta y}{\Delta x} \) is the slope of the line joining A and B in the Cartesian plane.

4. Consider the decision table below, with \( P(s_1) = p \):

\[
\begin{array}{c|cc}
 & s_1 & s_2 \\
\hline
A & 5 & 3 \\
B & 4 & 1 \\
C & 2 & 5 \\
\end{array}
\]

(a) For which value of \( p \) would the agent be indifferent between A and C?
(b) Plot the Bayes values for the strategies as \( p \) varies from 0 to 1.
(c) For which values of \( p \) are A, B, and C preferred, respectively, under the Bayes decision rule?

5. Alice sells drinks at a local market once every month. She can order stock to sell several drink types: a) hot chocolate; b) iced tea; c) lemonade; d) orange juice.

From past experience she knows that when she sells only one type of drink, on warm days her sales total for each type are: $10 on hot chocolate, $40 on iced tea, $30 on lemonade, and $40 on orange juice. On cool days, however, her sales totals are: $30 on hot chocolate, $0 on iced tea, $20 on lemonade, and $10 on orange juice.

She has to order her stock weeks in advance, long before she can predict the temperature on the day of the market.

(a) Produce a decision table for this problem.
(b) What proportion of drinks should she stock to maximise her guaranteed (i.e., minimum) sales total regardless of the temperature?
(c) Find the Bayes strategies for \( p = 0, \frac{1}{4}, \frac{1}{2}, \frac{3}{4}, 1 \).
(d) What is the least favourable probability distribution on warm and cool (not warm) days?
(e) Repeat the above analysis for the minimax Regret rule.
(f) Define the admissibility frontier for this problem.

6. Show that a strategy is admissible iff it is a Bayes strategy for some probability distribution.

7. Show that a Maximin strategy is always a Bayes strategy for some probability distribution.

8. Prove that for any two actions A and B, if A weakly dominates B, and all state probabilities are non-zero, then the Bayes decision rule will strictly prefer A over B.