NAME OF CANDIDATE: _____

STUDENT ID: _____

SIGNATURE: _____

THE UNIVERSITY OF NEW SOUTH WALES

2019 TERM 3

COMP6741: PARAMETERIZED AND EXACT COMPUTATION – Trial Exam

1. TIME ALLOWED – 2 hours
2. READING TIME – 10 minutes
3. THIS EXAMINATION PAPER HAS 3 PAGES
4. TOTAL NUMBER OF QUESTIONS – 3
5. TOTAL MARKS AVAILABLE – 100
6. ALL QUESTIONS ARE NOT OF EQUAL VALUE. MARKS AVAILABLE FOR EACH QUESTION ARE SHOWN IN THE EXAMINATION PAPER.
7. ALL ANSWERS MUST BE WRITTEN IN INK. PENCILS MAY BE USED ONLY FOR DRAWING, SKETCHING OR GRAPHICAL WORK.
8. THIS PAPER MAY NOT BE RETAINED BY THE CANDIDATE.

SPECIAL INSTRUCTIONS

9. ANSWER ALL QUESTIONS.
10. CANDIDATES MAY BRING TO THE EXAMINATION: any textbooks or notes (hard-copy), including annotated printed lecture notes, textbooks, handwritten and printed notes, UNSW approved calculator (but no other electronic devices).
11. THE FOLLOWING MATERIALS WILL BE PROVIDED: answer booklet

Your answers may rely on theorems, lemmas and results stated in the lecture notes and exercise sheets of this course.

1 ETH Lower Bound [15 marks]

Recall that a *feedback vertex set* of a multigraph $G = (V, E)$ is a set of vertices $S \subseteq V$ such that $G - S$ is acyclic.

Feedback Vertex Set (FVS)	
Input:	Multigraph $G = (V, E)$, integer k
Parameter:	k
Question:	Does G have a feedback vertex set of size at most k?

- Prove that Feedback Vertex Set has no $2^{o(k)}$ time algorithm if the Exponential Time Hypothesis is true.

2 3-SAT [40 marks]

Consider the Local-Search-3-Sat problem.

Local-Search-3-Sat (LS-3-Sat)	
Input:	A CNF formula F where each clause contains at most 3 literals, an assignment $\alpha : \mathsf{var}(F) \to \{0, 1\}$, and an integer k
Parameter:	k
Question:	Is there an assignment $\beta : \mathsf{var}(F) \to \{0, 1\}$ that differs with α on at most k variables and that satisfies F?

1. Design an $O^*(3^k)$ time algorithm for Local-Search-3-Sat. [15 marks]
2. Based on that algorithm, show that 3-Sat can be solved in $O^*(3^{n/2}) \subseteq O^*(1.7321^n)$ time, where $n = |\mathsf{var}(F)|$. [10 marks]
3. Assume there is a $O^*(2.792^k)$ time algorithm, called BK04, for LS-3-Sat.
   Design a $O^*(2.792^k)$ time algorithm for the following problem: [15 marks]

Find-LS-3-Sat	
Input:	A CNF formula F where each clause contains at most 3 literals, an assignment $\alpha : \mathsf{var}(F) \to \{0, 1\}$, and an integer k
Output:	An assignment $\beta : \mathsf{var}(F) \to \{0, 1\}$ that differs with α on at most k variables and that satisfies F, if there is one, and No otherwise.

3 Weighted 2-Regular Vertex Deletion [45 marks]

In a multigraph $G = (V, E)$, each edge contributes 1 to the degree of each of its endpoints; in case the edge connects a vertex to itself (*self-loop*), it contributes 2 to the degree of this vertex.

Consider the Weighted 2-Regular Vertex Deletion problem. A *2-regular multigraph* is a multigraph where every vertex has degree 2. That is, it is a disjoint union of cycles – this includes isolated 1-cycles (a vertex connected only to itself via one self-loop) and 2-cycles (a connected component with two vertices connected to each other by two edges).

Weighted 2-Regular Vertex Deletion (W2RVD)	
Input:	Multigraph $G = (V, E)$, a weight function $\omega : V \to \mathbb{N}^+$ assigning an integer $\omega(v) \geq 1$ to every vertex $v \in V$, and an integer k
Parameter:	k
Question:	Is there a set $S \subseteq V$ with weight $\sum_{v \in S} \omega(v)$ at most k such that $G - S$ is a 2-regular multigraph?

1. Design simplification rules that transform (G, ω, k) into an equivalent instance (G', ω', k') such that

   (a) G' has no vertex of degree at most 1, and [5 marks]

   (b) G' has no degree-2 vertex with a neighbor of degree 2 (and G' has no isolated 1-cycles, which is a special case). [10 marks]

2. Show that a multigraph with minimum degree at least 2 and no degree-2 vertex with a neighbor of degree 2 has average degree at least 2.4. [15 marks]

3. Show that there is a (possibly randomized) algorithm for W2RVD with running time $O^*(c^k)$ for some constant $c > 1$. [15 marks]

End of Paper