THE UNIVERSITY OF NEW SOUTH WALES

SEMESTER 2 2018

COMP6741: PARAMETERIZED AND EXACT COMPUTATION – Trial Exam

1. TIME ALLOWED – 3 hours
2. READING TIME – 10 minutes
3. THIS EXAMINATION PAPER HAS 3 PAGES
4. TOTAL NUMBER OF QUESTIONS – 4
5. TOTAL MARKS AVAILABLE – 100
6. ALL QUESTIONS ARE NOT OF EQUAL VALUE. MARKS AVAILABLE FOR EACH QUESTION ARE SHOWN IN THE EXAMINATION PAPER.
7. ALL ANSWERS MUST BE WRITTEN IN INK. EXCEPT WHERE THEY ARE EXPRESSLY REQUIRED, PENCILS MAY BE USED ONLY FOR DRAWING, SKETCHING OR GRAPHICAL WORK.
8. THIS PAPER MAY NOT BE RETAINED BY THE CANDIDATE.

SPECIAL INSTRUCTIONS

9. ANSWER ALL QUESTIONS.
10. CANDIDATES MAY BRING TO THE EXAMINATION: printed lecture notes, textbooks, handwritten and printed notes, UNSW approved calculator (but no other electronic devices).
11. THE FOLLOWING MATERIALS WILL BE PROVIDED: answer booklet

Your answers may rely on theorems, lemmas and results stated in the lecture notes and exercise sheets of this course.

1 ETH Lower Bound [10 marks]

Recall that a *feedback vertex set* of a multigraph $G = (V, E)$ is a set of vertices $S \subseteq V$ such that $G - S$ is acyclic.

FEEDBACK VERTEX SET	
Input:	Multigraph $G = (V, E)$, integer k
Parameter:	k
Question:	Does G have a feedback vertex set of size at most k?

- Prove that FEEDBACK VERTEX SET has no $2^{o(k)}$ time algorithm if the Exponential Time Hypothesis is true.

2 Weighted Cycle Vertex Deletion [30 marks]

Consider the WEIGHTED CYCLE VERTEX DELETION problem. A *cycle* is a 2-regular connected graph.

WEIGHTED CYCLE VERTEX DELETION	
Input:	Graph $G = (V, E)$, a weight function $\omega : V \to \mathbb{N}^+$ assigning an integer $\omega(v) \geq 1$ to every vertex $v \in V$, and an integer k
Parameter:	k
Question:	Is there a set $S \subseteq V$ with weight $\sum_{v \in S} \omega(v)$ at most k such that $G - S$ is a cycle?

1. Design simplification rules that transform (G, ω, k) into an equivalent instance (G', ω', k') such that
   (a) G' has no vertex of degree at most 1, and [5 marks]
   (b) G' has no degree-2 vertex with a neighbor of degree 2. [5 marks]
2. Show that a graph with minimum degree at least 2 and no two adjacent vertices of degree 2 has average degree at least 2.4. [10 marks]
3. Show that there is a (possibly randomized) algorithm for WEIGHTED CYCLE VERTEX DELETION with running time $O^*(c^k)$ for some constant $c > 1$. [10 marks]

3 Split Vertex Deletion [30 marks]

Recall that an *independent set* in a graph $G = (V, E)$ is a subset of vertices $S \subseteq V$ such that every two vertices from S are non-adjacent in G. A graph $G = (V, E)$ is a *split graph* if V can be partitioned into an independent set I and a clique C. Consider the SPLIT VERTEX DELETION problem.

SPLIT VERTEX DELETION	
Input:	Graph $G = (V, E)$, integer k
Parameter:	k
Question:	Is there a set $S \subseteq V$ of size k such that $G - S$ is a split graph?

- Design an $O^*(2^k)$ time algorithm for the Split Vertex Deletion problem.

It is known that the problem of deciding whether an input graph $G = (V, E)$ is a split graph can be solved in $O(|V| + |E|)$ time, and you may use this algorithm as a black box.

It is recommended to use iterative compression and answer the following questions:

1. Formulate a disjoint version of the problem and show that a polynomial time algorithm for the disjoint version implies an $O^*(2^k)$ time algorithm for Split Vertex Deletion. [10 marks]
2. Show that a split graph on n vertices has only a polynomial number of partitions of the vertex set into a clique and an independent set. [10 marks]
3. Show that the disjoint version of the problem can be solved in polynomial time. [10 marks]

4 3-SAT [30 marks]

Consider the Local-Search-3-Sat problem.

Local-Search-3-Sat (LS-3-Sat)	
Input:	A CNF formula F where each clause contains at most 3 literals, an assignment $\alpha : \mathsf{var}(F) \to \{0,1\}$, and an integer k
Parameter:	k
Question:	Is there an assignment $\beta : \mathsf{var}(F) \to \{0,1\}$ that differs with α on at most k variables and that satisfies F?

1. Design an $O^*(3^k)$ time algorithm for Local-Search-3-Sat. [10 marks]
2. Based on that algorithm, show that 3-Sat can be solved in $O^*(3^{n/2}) \subseteq O^*(1.7321^n)$ time, where $n = |\mathsf{var}(F)|$. [10 marks]
3. Assume there is a $O^*(2.792^k)$ time algorithm, called BK04, for LS-3-Sat.
   Design a $O^*(2.792^k)$ time algorithm for the following problem: [10 marks]

Find-LS-3-Sat	
Input:	A CNF formula F where each clause contains at most 3 literals, an assignment $\alpha : \mathsf{var}(F) \to \{0,1\}$, and an integer k
Output:	An assignment $\beta : \mathsf{var}(F) \to \{0,1\}$ that differs with α on at most k variables and that satisfies F, if there is one, and No otherwise.

End of Paper