COMP4418: Knowledge Representation and Reasoning—Solutions to Exercise 1 Propositional Logic

1. (i) $(\neg Ja \wedge \neg Jo) \rightarrow T$
   Where:
   Ja: *Jane is in town*
   Jo: *John is in town*
   T: *we will play tennis*

   (ii) $R \vee \neg R$
   Where:
   R: *it will rain today*

   (iii) $\neg S \rightarrow \neg P$
   Where:
   S: *you study*
   P: *you will pass this course*

   (iv) $D \rightarrow (B \vee S)$
   Where:
   D: *I ate dinner*
   B: *I drink bubble tea*
   S: *I drink soft drink*

   (v) $(V \wedge D) \rightarrow (R \wedge \neg F)$
   Where:
   V: *80% of adults are fully vaccinated*
   D: *COVID-19 cases begin to drop*
   R: *lockdown restrictions are eased*
   F: *international flights immediately resume*

2. (i) $P \rightarrow Q$
   $\neg P \vee Q$ (remove $\rightarrow$)

   (ii) $(P \rightarrow \neg Q) \rightarrow R$
   $\neg(\neg P \vee \neg Q) \vee R$ (remove $\rightarrow$)
   $(\neg\neg P \wedge \neg\neg Q) \vee R$ (De Morgan)
   $(P \wedge Q) \vee R$ (Double Negation)
   $(P \vee R) \wedge (Q \vee R)$ (Distribute $\vee$ over $\wedge$)

   (iii) $\neg(P \wedge \neg Q) \rightarrow (\neg R \vee \neg Q)$
   $\neg\neg(P \wedge \neg Q) \vee (\neg R \vee \neg Q)$ (remove $\rightarrow$)
   $(P \wedge \neg Q) \vee (\neg R \vee \neg Q)$ (Double Negation)
   $(P \vee \neg R \vee \neg Q) \wedge (\neg Q \vee \neg R \vee \neg Q)$ (Distribute $\vee$ over $\wedge$)
   This can be further simplified to: $((P \vee \neg R \vee \neg Q) \wedge (\neg Q \vee \neg R)$
   And in fact this cab be simplified to $\neg Q \vee \neg R$ since $(\neg Q \vee \neg R) \vdash (P \vee \neg R \vee \neg Q)$

   (iv) $(\neg P \rightarrow Q) \rightarrow (Q \rightarrow \neg R)$
   $\neg(\neg P \rightarrow Q) \vee (Q \rightarrow \neg R)$ (remove $\rightarrow$)

$\neg(P \lor Q) \lor (\neg Q \lor \neg R)$ (remove $\to$)
$(\neg P \land \neg Q) \lor (\neg Q \lor \neg R)$ (De Morgan)
$(\neg P \lor \neg Q \lor \neg R) \land (\neg Q \lor \neg R)$ (Distribute $\lor$ over $\land$)

(v) $\neg(\neg P \lor Q) \lor (\neg R \to S)$
$\neg(\neg P \lor Q) \lor (\neg\neg R \lor S)$ (remove $\to$)
$(\neg\neg P \land \neg Q) \lor (\neg\neg R \lor S)$ (De Morgan)
$(P \land \neg Q) \lor (R \lor S)$ (Double Negation)
$(P \lor R \lor S) \land (\neg Q \lor R \lor S)$ (Distribute $\lor$ over $\land$)

3. (i)

P	Q	$P \to Q$	$\neg Q$	$\neg P$
T	T	T	F	F
T	F	F	T	F
F	T	T	F	T
F	F	T	T	T

In all rows where both $P \to Q$ and $\neg Q$ are true, $\neg P$ is also true. Therefore, inference is valid.

(ii)

P	Q	$\neg P$	$\neg Q$	$P \to Q$	$\neg Q \to \neg P$
T	T	F	F	T	T
T	F	F	T	F	F
F	T	T	F	T	T
F	F	T	T	T	T

In all rows where $P \to Q$ is true, $\neg Q \to \neg P$ is also true. Therefore, inference is valid.

(iii)

P	Q	R	$P \to Q$	$Q \to R$	$P \to R$
T	T	T	T	T	T
T	T	F	T	F	F
T	F	T	F	T	T
T	F	F	F	T	F
F	T	T	T	T	T
F	T	F	T	F	T
F	F	T	T	T	T
F	F	F	T	T	T

In all rows where both $P \to Q$ and $Q \to R$ are true, $P \to R$ is also true. Therefore, inference is valid.

(iv)

P	Q	R	$Q \land R$	$P \to Q$	$P \to R$	$P \to (Q \land R)$
T	T	T	T	T	T	T
T	T	F	F	T	F	F
T	F	T	F	F	T	F
T	F	F	F	F	F	F
F	T	T	T	T	T	T
F	T	F	F	T	T	T
F	F	T	F	T	T	T
F	F	F	F	T	T	T

In all rows where both $P \to Q$ and $P \to R$ are true, $P \to (Q \land R)$ is also true. Therefore, inference is valid.

(v)

P	Q	R	$P \wedge Q$	$Q \to R$	$P \to (Q \to R)$	$(P \wedge Q) \to R$
T	T	T	T	T	T	T
T	T	F	T	F	F	F
T	F	T	F	T	T	T
T	F	F	F	T	T	T
F	T	T	F	T	T	T
F	T	F	F	F	T	T
F	F	T	F	T	T	T
F	F	F	F	T	T	T

In all rows where $P \to (Q \to R)$ is true, $(P \wedge Q) \to R$ is also true. Therefore, inference is valid.

4. (i) CNF$(P \to Q)$
$\equiv \neg P \vee Q$

CNF$(\neg Q)$
$\equiv \neg Q$

CNF$(\neg\neg P)$
$\equiv P$ (Double Negation)

Proof:

1.	$\neg P \vee Q$	(Hypothesis)
2.	$\neg Q$	(Hypothesis)
3.	P	(Negation of Conclusion)
4.	Q	1, 3 Resloution
5.	$\Box$	2, 4 Resloution

(ii) CNF$(P \to Q)$
$\equiv \neg P \vee Q$

CNF$(\neg(\neg Q \to \neg P))$
$\equiv \neg(\neg\neg Q \vee \neg P)$ (Remove $\to$)
$\equiv \neg(Q \vee \neg P)$ (Double Negation)
$\equiv \neg Q \wedge \neg\neg P$ (De Morgan)
$\equiv \neg Q \wedge P$ (Double Negation)

Proof:

1.	$\neg P \vee Q$	(Hypothesis)
2.	$\neg Q$	(Negation of Conclusion)
3.	P	(Negation of Conclusion)
4.	$\neg P$	1, 2 Resolution
5.	$\Box$	3, 4 Resolution

(iii) $P \to Q, Q \to R \vdash P \to R$
CNF$(P \to Q)$
$\equiv \neg P \vee Q$

CNF$(Q \to R)$
$\equiv \neg Q \vee R$

CNF($\neg(P \to R)$)
$\equiv \neg(\neg P \lor R)$ (Remove $\to$)
$\equiv \neg\neg P \land \neg R$ (De Morgan)
$\equiv P \land \neg R$ (Double Negation)

Proof:

1.	$\neg P \lor Q$	(Hypothesis)
2.	$\neg Q \lor R$	(Hypothesis)
3.	P	(Negation of Conclusion)
4.	$\neg R$	(Negation of Conclusion)
5.	Q	1, 3 Resolution
6.	R	2, 5 Resolution
7.	$\square$	4, 6 Resolution

(iv) $P \to Q, P \to R \vdash P \to (Q \land R)$

CNF($P \to Q$)
$\equiv \neg P \lor Q$

CNF($P \to R$)
$\equiv \neg P \lor R$

CNF($\neg(P \to (Q \land R))$)
$\equiv \neg(\neg P \lor (Q \land R))$ (Remove $\to$)
$\equiv \neg\neg P \land \neg(Q \land R)$ (De Morgan)
$\equiv P \land (\neg Q \lor \neg R)$ (Double Negation, De Morgan)

Proof:

1.	$\neg P \lor Q$	(Hypothesis)
2.	$\neg P \lor R$	(Hypothesis)
3.	P	(Negation of Conclusion)
4.	$\neg Q \lor \neg R$	(Negation of Conclusion)
5.	Q	1, 3 Resolution
6.	R	2, 3 Resolution
7.	$\neg R$	4, 5 Resolution
8.	$\square$	6, 7 Resolution

(v) $P \to (Q \to R) \vdash (P \land Q) \to R$

CNF($P \to (Q \to R)$)
$\equiv P \to (\neg Q \lor R)$ (Remove $\to$)
$\equiv \neg P \lor (\neg Q \lor R)$ (Remove $\to$)
$\equiv \neg P \lor \neg Q \lor R$

CNF($\neg((P \land Q) \to R)$)
$\equiv \neg(\neg(P \land Q) \lor R)$ (Remove $\to$)
$\equiv \neg\neg(P \land Q) \land \neg R$ (De Morgan)
$\equiv (P \land Q) \land \neg R$ (Double Negation)
$\equiv P \land Q \land \neg R$

Proof:

1.	$\neg P \vee \neg Q \vee R$	(Hypothesis)
2.	P	(Negation of Conclusion)
3.	Q	(Negation of Conclusion)
4.	$\neg R$	(Negation of Conclusion)
5.	$\neg Q \vee R$	1, 2 Resolution
6.	R	3, 5 Resolution
7.	$\Box$	4, 6 Resolution

5. (i) $((P \vee Q) \wedge \neg P) \rightarrow Q$

P	Q	$\neg P$	$P \vee Q$	$(P \vee Q) \wedge \neg P$	$((P \vee Q) \wedge \neg P) \rightarrow Q$
T	T	F	T	F	T
T	F	F	T	F	T
F	T	T	T	T	T
F	F	T	F	F	T

Last column is always true no matter what truth assignment to the atoms P and Q. Therefore $((P \vee Q) \wedge \neg P) \rightarrow Q$ is a tautology.

(ii) $((P \rightarrow Q) \wedge \neg(P \rightarrow R)) \rightarrow (P \rightarrow Q)$

P	Q	R	$P \rightarrow Q$	$\neg(P \rightarrow R)$	$(P \rightarrow Q) \wedge \neg(P \rightarrow R)$	$((P \rightarrow Q) \wedge \neg(P \rightarrow R)) \rightarrow (P \rightarrow Q)$
T	T	T	T	F	F	T
T	T	F	T	T	T	T
T	F	T	F	F	F	T
T	F	F	F	T	F	T
F	T	T	T	F	F	T
F	T	F	T	F	F	T
F	F	T	T	F	F	T
F	F	F	T	F	F	T

Last column is always true no matter what truth assignment to the atoms P, Q and R. Therefore $((P \rightarrow Q) \wedge \neg(P \rightarrow R)) \rightarrow (P \rightarrow Q)$ is a tautology.

(iii) $\neg(\neg P \wedge P) \wedge P$

P	$\neg P$	$\neg P \wedge P$	$\neg(\neg P \wedge P)$	$\neg(\neg P \wedge P) \wedge P$
T	F	F	T	T
F	T	F	T	F

Last column is not always true. Therefore $\neg(\neg P \wedge P) \wedge P$ is not a tautology.

(iv) $(P \vee Q) \rightarrow \neg(\neg P \wedge \neg Q)$

P	Q	$\neg P$	$\neg Q$	$P \vee Q$	$\neg P \wedge \neg Q$	$\neg(\neg P \wedge \neg Q)$	$(P \vee Q) \rightarrow \neg(\neg P \wedge \neg Q)$
T	T	F	F	T	F	T	T
T	F	F	T	T	F	T	T
F	T	T	F	T	F	T	T
F	F	T	T	F	T	F	T

(v) $(P \vee Q) \wedge \neg(P \wedge Q)$

P	Q	$P \vee Q$	$P \wedge Q$	$\neg(P \wedge Q)$	$(P \vee Q) \wedge \neg(P \wedge Q)$
T	T	T	T	F	F
T	F	T	F	T	T
F	T	T	F	T	T
F	F	F	F	T	F

Last column is not always true. Therefore $(P \vee Q) \wedge \neg(P \wedge Q)$ is not a tautology.

6. (i) CNF$(\neg(((P \vee Q) \wedge \neg P) \rightarrow Q)) \equiv \neg(\neg((P \vee Q) \wedge \neg P) \vee Q)$ (Remove $\rightarrow$)
$\equiv \neg\neg((P \vee Q) \wedge \neg P) \wedge \neg Q$ (DeMorgan)
$\equiv (P \vee Q) \wedge \neg P) \wedge \neg Q$ (Double Negation)

Proof:

1.	$P \vee Q$	(Negated Conclusion)
2.	$\neg P$	(Negated Conclusion)
3.	$\neg Q$	(Negated Conclusion)
4.	Q	1, 2 Resolution
5.	$\square$	3, 4 Resolution

Therefore $\neg(((P \vee Q) \wedge \neg P) \rightarrow Q)$ is a tautology.

(ii) CNF$(\neg(((P \rightarrow Q) \wedge \neg(P \rightarrow R)) \rightarrow (P \rightarrow Q)))$
$\equiv \neg(\neg((\neg P \vee Q) \wedge \neg(\neg P \vee R)) \vee (\neg P \vee Q))$ (Remove $\rightarrow$)
$\equiv \neg\neg((\neg P \vee Q) \wedge \neg(\neg P \vee R)) \wedge \neg(\neg P \vee Q)$ (De Morgan)
$\equiv (\neg P \vee Q) \wedge (\neg\neg P \wedge \neg R) \wedge (\neg\neg P \wedge \neg Q)$ (Double Negation and De Morgan)
$\equiv (\neg P \vee Q) \wedge (P \wedge \neg R) \wedge (P \wedge \neg Q)$ (Double Negation)

Proof:

1.	$\neg P \vee Q$	(Negated Conclusion)
2.	P	(Negated Conclusion)
3.	$\neg R$	(Negated Conclusion)
4.	$\neg Q$	(Negated Conclusion)
5.	Q	1, 2 Resolution
6.	$\square$	4, 5 Resolution

Therefore $((P \rightarrow Q) \wedge \neg(P \rightarrow R)) \rightarrow (P \rightarrow Q)$ is a tautology.

(iii) CNF$(\neg(\neg(\neg P \wedge P) \wedge P))$
$\equiv \neg\neg(\neg P \wedge P) \vee \neg P$ (De Morgan)
$\equiv (\neg P \wedge P) \vee \neg P$ (Double Negation)
$\equiv (\neg P \vee \neg P) \wedge (P \vee \neg P)$ (Distribute $\wedge$ over $\vee$)
$\equiv \neg P$ (Can simplify to this by removing repetition and tautologies)

Proof:

1.	$\neg P$	(Negated Conclusion)

Cannot obtain empty clause using resolution so $\neg(\neg P \wedge P) \wedge P$ is not a tautology.

(iv) CNF($\neg((P \vee Q) \rightarrow \neg(\neg P \wedge \neg Q))$)
$\equiv \neg(\neg(P \vee Q) \vee \neg(\neg P \wedge \neg Q))$ (Remove $\rightarrow$)
$\equiv \neg\neg(P \vee Q) \wedge \neg\neg(\neg P \wedge \neg Q))$ (De Morgan)
$\equiv (P \vee Q) \wedge (\neg P \wedge \neg Q))$ (Double Negation)

Proof:

1.	$(P \vee Q)$	(Negated Conclusion)
2.	$\neg Q$	(Negated Conclusion)
3.	$\neg P$	(Negated Conclusion)
4.	Q	1, 2 Resolution
5.	$\square$	3, 4, Resolution

Therefore $(P \vee Q) \rightarrow \neg(\neg P \wedge \neg Q)$ is a tautology.

(v) CNF($\neg((P \vee Q) \wedge \neg(P \wedge Q))$)
$\equiv \neg(P \vee Q) \vee \neg\neg(P \wedge Q)$ (De Morgan)
$\equiv \neg(P \vee Q) \vee (P \wedge Q)$ (Double Negation)
$\equiv (\neg P \wedge \neg Q) \vee (P \wedge Q)$ (De Morgan)
$\equiv (\neg P \vee P) \wedge (\neg P \vee Q) \wedge (\neg Q \vee P) \wedge (\neg Q \vee Q)$ (Distribution)
$\equiv (\neg P \vee Q) \wedge (P \vee \neg Q)$ (Removal of tautologies)

Proof:

1.	$(\neg P \vee Q)$	(Negated Conclusion)
2.	$(P \vee \neg Q)$	(Negated Conclusion)

Cannot obtain empty clause using resolution so $(P \vee Q) \wedge \neg(P \wedge Q)$ is not a tautology.

7. P: *I will listen to the album "SOUR" by Olivia Rodrigo*
Q: *I will watch another episode of The Queen's Gambit*
Truth table:
$P \vee Q, \neg Q \models \neg P$

P	Q	$P \vee Q$	$\neg Q$	$\neg P$
T	T	T	F	F
T	F	T	T	F
F	T	T	F	T
F	F	F	T	T

This inference is not valid as $\neg P$ is not always true when $(P \vee Q)$ and $\neg Q$ are both true.

Resolution:
$P \vee Q, \neg Q \vdash \neg P$

CNF($\neg\neg P$)
$\equiv P$ (Double Negation)

Proof:

1.	$P \vee Q$	(Hypothesis)
2.	$\neg Q$	(Hypothesis)
3.	P	(Negated Conclusion)
4.	P	1, 2 Resolution

This inference is not valid as we cannot derive the empty clause using resolution.

8. B: *I will drink too much bubble tea*
S: *I feel sick*
Truth table:
$(B \to S) \vee (S \to B)$

B	S	$B \to S$	$S \to B$	$(B \to S) \vee (S \to B)$
T	T	T	T	T
T	F	F	T	T
F	T	T	F	T
F	F	T	T	T

Last column is always true. Therefore the statement is a tautology.

Resolution:
$\vdash (B \to S) \vee (S \to B)$

CNF$(\neg((B \to S) \vee (S \to B)))$
$\equiv \neg((\neg B \vee S) \vee (\neg S \vee B))$ (Remove $\to$)
$\equiv \neg(\neg B \vee S) \wedge \neg(\neg S \vee B)$ (De Morgan)
$\equiv (\neg\neg B \wedge \neg S) \wedge (\neg\neg S \wedge \neg B)$ (De Morgan)
$\equiv (B \wedge \neg S) \wedge (S \wedge \neg B)$ (Double Negation)

Proof:

1.	$B \wedge \neg S$	(Negated Conclusion)
2.	$S \wedge \neg B$	(Negated Conclusion)
3.	$\square$	(1, 2 Resolution)

Therefore the sentence is a tautology.