Assignment 3
COMP6741: Parameterized and Exact Computation

2017, Semester 2

This is an individual assignment. For the solutions to this assignment, you may rely on all theorems, lemmas, and results from the lecture notes. If any other works (articles, books, Wikipedia entries, lecture notes from other courses, etc.) inspired your solutions, please cite them and give a list of references at the end.

If you have questions about this assignment, please post them to the Forum.

Due date. This assignment is due on Wednesday, 11 October 2017, at 23.59 AEST. Submitting x days after the deadline, with $x > 0$, reduces the grade by $20 \cdot x$ per cent.

How to submit. Submit a PDF with your solutions using the command

give cs6741 a3 <mysolution.pdf>

from the CSE network, or use the WebCMS3 frontend for `give`. The first page should contain your name and Student ID.

Recall that a graph H is an *induced subgraph* of a graph G if H can be obtained from G by a a sequence of vertex deletions. Recall that a graph H is a *subgraph* of a graph G if H can be obtained from G by a sequence of vertex deletions and edge deletions.

The *contraction* of an edge $uv \in E$ in a graph G is an operation that replaces the vertices u and v by a new vertex w_{uv} with $N(w_{uv}) = (N_G(u) \cup N_G(v)) \setminus \{u, v\}$. A graph H is a *minor* of a graph G if H can be obtained from G by a a sequence of vertex deletions, edge deletions, and edge contractions.

1. Recall that a *cycle* in a graph G is a 2-regular connected subgraph of G. Consider the Long Cycle problem.

Long Cycle (LC)	
Input:	A graph $G = (V, E)$ and an integer k
Parameter:	k
Question:	Does G have a cycle of length at least k?

The goal will be to design a (possibly randomized) FPT algorithm for LC. Note that the problem statement asks for a cycle of length at least k (and not of length exactly k) and it is not possible, in general, to bound the length of a shortest cycle of length at least k by any function of k.

(a) Show that the following holds: if the graph G has a cycle of length at least $2k$ and G' is a graph obtained from G by contracting an edge, then G' has a cycle of length at least k. [10 points]

(b) For some constant c, design a $O^*(c^k)$ time (randomized) algorithm for the variant of LC that asks whether G has a cycle of length at least k and at most $2k$. [20 points]

(c) For some constant d, design a $O^*(d^k)$ time (randomized) algorithm for LC. [10 points]

2. A graph G is isomorphic to another graph H if H can be obtained from G by renaming vertices. Consider the following parameterized problem:

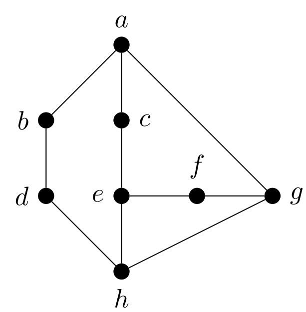
SUBGRAPH	
Input:	A graph $G = (V, E)$ and a graph $H = (V', E')$
Parameter:	$\lvert V' \rvert$
Question:	Does G have a subgraph isomorphic to H?

- Show that SUBGRAPH is W[1]-hard. [10 points]

3. The next set of questions is about treewidth and graph minors. Prove the following three statements.

   (a) If H is a minor of a graph G, then the treewidth of H is at most the treewidth of G. [15 points]

   (b) The following graph has treewidth 3:



[10 points]

   (c) The TW-CLIQUE MINOR problem is fixed-parameter tractable:

TW-CLIQUE MINOR	
Input:	A graph G, a tree decomposition (T, γ) of G, and an integer r
Parameter:	the width of the tree decomposition (T, γ)
Question:	Does G have a complete graph on r vertices as a minor?

[25 points]