COMP4418: Knowledge Representation and Reasoning

Logic and Prolog

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Logic and Prolog

- Prolog stands for programming in logic
- How does the implementation of Prolog relate to logic?
- Prolog is based on resolution theorem proving in first-order logic
- In this lecture we will look at the relationship between automated reasoning in first-order logic and Prolog
- References:
  - Ivan Bratko, Prolog Programming for Artificial Intelligence, Addison-Wesley, 2001. (Chapter 2)

Overview

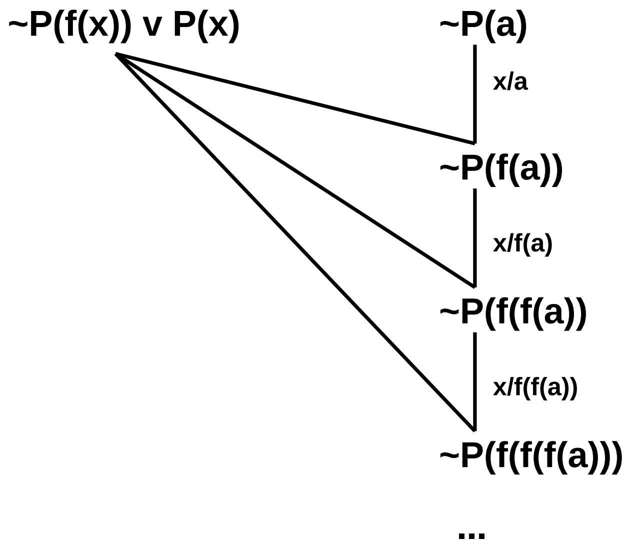
- Problems
- Undecidability of first-order logic
- Horn Clauses
- SLD Resolution
- Prolog
- Back Chaining
- Forward Chaining
- Negation as Failure
- Conclusion

Resolution — Problem 1

- We have seen that the resolution rule is sound:
  If $\Gamma \vdash \phi$, then $\Gamma \models \phi$
- However, the resolution rule is not complete in general:
  $\{\neg P\} \models \neg P \vee \neg Q$ but cannot show this using resolution ($\{\neg P\} \vdash \neg P \vee \neg Q$)
- Resolution is sound and complete when used as a refutation system though:
  $\Gamma \vdash \Box$ if and only if $\Gamma \models \Box$
- Therefore, resolution should be used as a refutation system as we have done so far

Resolution — Problem 2

- $KB = \{P(f(x) \rightarrow P(x)\}$
- $Q = P(a)$?
- Obviously $KB \not\models Q$
- However, let us attempt to show this using resolution



Undecidability of First-Order Logic

- Can we determine in general when this problem will arise?
- **Answer:** no!
- There is no general procedure

  if (KB unsatisfiable)
  return Yes; **Halt**
  else return No; **Halt**

- Resolution is refutation complete so if KB is unsatisfiable search tree will contain empty clause somewhere
- Can find empty clause using breadth-first search (why?) but if the search tree does not contain the empty clause the search may go on forever
- Even in the propositional case (which is decidable), complexity of resolution is $O(2^n)$

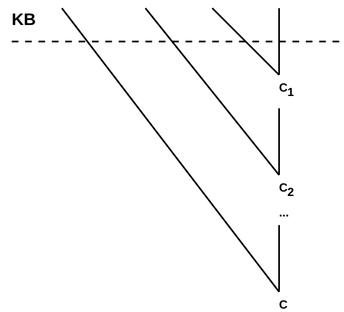
Horn Clauses

Idea: use less expressive language

- Review
  - Literals — atomic sentence or its negation
  - Clause — disjunction of literals
- Horn Clause – at most one positive literal (e.g., $\neg P \vee Q$, $P \vee \neg Q \vee R \vee S$)
  - Essentially represents a formula of the form $A_1 \wedge \ldots \wedge A_n \rightarrow C$
  - That is, if A_1 and ... and A_n, then C
- Definite (Positive) Clause – exactly one positive literal
- Negative Clause – no positive literals

SLD Resolution — $\vdash_{SLD}$

- Selected literals Linear form Definite clauses resolution
- SLD derivation of a clause C from a set of clauses KB is a sequence of clauses such that
  1. First clause of sequence comes from KB
  2. Each intermediate clause C_i is derived by resolving the previous clause C_{i-1} and a clause from KB
  3. The last clause in the sequence is C



- For set of Horn clauses KB: $KB \vdash \Box$ if and only if $KB \vdash_{SLD} \Box$

Prolog

- Horn clauses in first-order logic (facts and rules)
- SLD resolution
- Depth-first search strategy with backtracking
- User control
  - Ordering of predicates in Prolog database (facts and rules)
  - Ordering of subgoals in body of a rule
  - Cut (!) operator
  - Negation as failure
- That is, Prolog is a restricted form of first-order logic (Horn clauses) and puts more control of the theorem proving process into the hands of the programmer allowing them to use problem-specific knowledge to reduce search

Backward Chaining

(Brachman & Levesque) Show whether Horn knowledge base satisfiable

- Goal driven
- Start with hypothesis and work backwards using rules in knowledge base to easily confirmed findings
- Check satisfiabilty of set of Horn clauses:

prove($Q_1 \wedge \ldots \wedge Q_n$) {
  if $n = 0$ **return** yes % empty clause
  for each $R \in KB$ **do**
    if $R = Q_1 \leftarrow G_1 \wedge \ldots \wedge G_m$ and prove($G_1 \wedge \ldots \wedge G_m \wedge Q_2 \wedge \ldots \wedge Q_n$)
    then return yes
  return no }

- Depth-first, left-right, backward chaining
- Strategy applied by Prolog

Forward Chaining

(Brachman & Levesque) Determine whether Horn knowledge base entails query: $KB \models Q$

- Data driven
- 1. **if** Q marked `solved` **then return** yes
  2. **if** $G \leftarrow G_1 \wedge \ldots \wedge G_m \in KB$ **and** $G_1, \ldots, G_m$ marked `solved` and G not marked `solved`
     then mark G `solved`; goto 1
     else return no

Negation as Failure

- Prolog does not implement classical negation
- Prolog `not` is known as negation as failure
- ```
  not(G) :- G, !, fail. % If G succeeds return no
  not(G).               % else return yes
  ```
- $KB \vdash \texttt{not}(G)$ — cannot prove G
- $KB \vdash \neg G$ — can prove $\neg G$
- They are not the same
- Negation as failure is finite failure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# Conclusion

- First-order logic is an expressive formal language and allows for powerful reasoning
- Theorem proving is undecidable in general
- Other options:
  - Search heuristics (ordering of predicates, subgoals; depth-first search)
  - Sacrifice expressivity (e.g., Horn clauses although still undecidable in first-order case)
  - User control (cut operator)
- Prolog is based on SLD resolution in first-order Horn logic and allows programmer to use knowledge about domain to control search
- Blend of theory and pragmatics