COMP4418: Knowledge Representation and Reasoning—Exercise Set 2 First-Order Logic

1. Translate the following first-order sentences into English

   (i) $\forall x.(bird(x) \rightarrow flies(x))$

   (ii) $\forall x.(person(x) \rightarrow \neg flies(x))$

   (iii) $\forall x.\exists y.(person(x) \rightarrow mother(y, x))$

   (iv) $\exists x.\forall y.(person(x) \wedge mother(x, y))$

   (v) $\exists x.(person(x) \wedge \forall y.(person(y) \rightarrow \neg mother(x, y)))$

   Where:

   $bird(x)$ means "x is a bird"
   $flies(x)$ means "x flies"
   $person(x)$ means "x is a person"
   $mother(x, y)$ means "x is the mother of y"

2. Convert the following English sentences into sentences of first-order logic:

   (i) All cats are mammals.

   (ii) No cat is a reptile.

   (iii) All computer scientists like some operating system.

   (iv) Every student engages in at least one hobby.

   (v) There exists a handsome student who writes up all of these questions.

   (vi) Everyone who teaches a student is taught by someone else.

   Use meaningful predicate names or state the scheme of abbreviation that you are using.

3. Convert the following first-order sentences into conjunctive normal form:

   (i) $\forall x.(bird(x) \rightarrow flies(x))$

   (ii) $\neg\exists x.(pig(x) \wedge flies(x))$

   (iii) $\forall x.(student(x) \rightarrow \exists y.(hobby(y) \wedge engages(x, y)))$

   (iv) $\forall x.((\forall y.student(y) \rightarrow teaches(x, y)) \rightarrow (\exists y.teaches(y, x)))$

   (v) $\exists x.\forall y.\forall z.(person(x) \wedge ((likes(x, y) \wedge y \neq z) \rightarrow \neg likes(x, z))$

4. Determine whether the following are valid inferences in first-order logic using resolution.

   (i) $\forall x.(P(x) \rightarrow Q(x)) \vdash \forall y.(\neg Q(y) \rightarrow \neg P(y))$

   (ii) $\forall x.(P(x) \rightarrow Q(x)) \vdash \forall x.(\neg Q(x) \rightarrow \neg P(x))$

   (iii) $\forall x.(P(x) \rightarrow Q(x)), P(a) \vdash Q(a)$

(iv) $\forall x.(P(x) \to Q(x)), \exists x.P(x) \vdash \exists x.Q(x)$

(v) $\forall x.(P(x) \to Q(x)), \forall x.(Q(x) \to R(x)) \vdash \forall x.(P(x) \to R(x))$

5. Consider the following three sentences:

   (A) There is a computer scientist who likes every operating system

   (B) Linux is an operating system

   (C) Someone likes Linux

   We wish to investigate the relationship among these three sentences:

   (i) Write a formula in first-order logic expressing each of the given facts. Call them A, B and C.

   (ii) Write the set of clauses corresponding to A, B and $\neg C$.

   (iii) Derive the empty clause from this set of clauses using resolution.

   (iv) Is there an SLD resolution of the empty clause? Why or why not?

   (v) Explain what entailment relation the resolution derivation shows among the three sentences.

6. Consider the following three sentences:

   (A) All rock songs and pop songs I have listened to are amazing.

   (B) There is at least one song that isn't amazing.

   (C) There is at least one song I have listened to that is neither a rock song nor a pop song.

   We wish to investigate the relationship among these three sentences:

   (i) Write a formula in first-order logic expressing each of the given facts. Call them A, B and C.

   (ii) Write the set of clauses corresponding to A, B and $\neg C$.

   (iii) Attempt to resolve this set of clauses.

   (iv) Is there an SLD resolution of the empty clause? Why or why not?

   (v) Explain what entailment relation the resolution derivation shows among the three sentences.