COMP4418: Knowledge Representation—Solutions to Exercise Set 2
First-Order Logic

1. (i) All birds fly
   (If an object x is a bird, then it flies.)

   (ii) All people cannot fly

   (iii) Everyone has a mother

   (iv) There is someone who is everyone's mother

   (v) There is someone who is no one's mother

2. (i) $\forall x.(cat(x) \rightarrow mammal(x))$

   (ii) $\neg\exists x.(cat(x) \wedge reptile(x))$
   or, equivalently, $\forall x.(cat(x) \rightarrow \neg reptile(x))$

   (iii) $\forall x.\exists y.(computer\_scientist(x) \rightarrow likes(x, y))$

   (iv) $\forall x.(student(x) \rightarrow \exists y.(hobby(y) \wedge engages(x, y)))$

   (v) $\exists x.((handsome(x) \wedge student(x)) \wedge \forall y.(question(y) \rightarrow writes(x, y)))$

   (vi) $\forall x.((\forall y.student(y) \rightarrow teaches(x, y)) \rightarrow (\exists y.teaches(y, x)))$

3. (i) CNF($\forall x.(bird(x) \rightarrow flies(x))$)
   $\equiv \forall x.(\neg bird(x) \vee flies(x))$ (Remove $\rightarrow$)
   $\equiv \neg bird(x) \vee flies(x)$ (Drop $\forall$))

   (ii) CNF($\neg\exists x.(pig(x) \wedge flies(x))$)
   $\equiv \forall x.\neg(pig(x) \wedge flies(x))$ (Move negation inwards)
   $\equiv \forall x.(\neg pig(x) \vee \neg flies(x))$ (De Morgan)
   $\equiv \neg pig(x) \vee \neg flies(x)$ (Drop $\forall$)

   (iii) CNF($\forall x.(student(x) \rightarrow \exists y.(hobby(y) \wedge engages(x, y)))$)
   $\equiv \forall x.(\neg student(x) \vee \exists y.(hobby(y) \wedge engages(x, y)))$ (Remove $\rightarrow$)
   $\equiv \forall x.(\neg student(x) \vee (hobby(f(x)) \wedge engages(x, f(x))))$ (Skolemize—f is Skolem function)
   $\equiv \neg student(x) \vee (hobby(f(x)) \wedge engages(x, f(x)))$ (Drop $\forall$)
   $\equiv (\neg student(x) \vee hobby(f(x))) \wedge (\neg student(x) \vee engages(x, f(x)))$ (Distribute $\vee$ over $\wedge$)

   (iv) CNF($\forall x.((\forall y.student(y) \rightarrow teaches(x, y)) \rightarrow (\exists y.teaches(y, x)))$)
   $\equiv \forall x.(\neg(\forall y.\neg student(y) \vee teaches(x, y)) \vee (\exists y.teaches(y, x)))$ (Remove $\rightarrow$)
   $\equiv \forall x.((\exists y.\neg(\neg student(y) \vee teaches(x, y))) \vee (\exists y.teaches(y, x)))$ (Move negation inwards)
   $\equiv \forall x.((\exists y.\neg\neg student(y) \wedge \neg teaches(x, y)) \vee (\exists y.teaches(y, x)))$ (De Morgan)
   $\equiv \forall x.((\exists y.student(y) \wedge \neg teaches(x, y)) \vee (\exists z.teaches(z, x)))$ (Standardize variables)
   $\equiv \forall x.((student(f(x)) \wedge \neg teaches(x, f(x))) \vee teaches(g(x), x))$ (Skolemize—f and g are Skolem functions)

$\equiv (student(f(x)) \wedge \neg teaches(x, f(x))) \vee teaches(g(x), x)$ (Drop $\forall$)
$\equiv (student(f(x)) \vee teaches(g(x), x)) \wedge (\neg teaches(x, f(x)) \vee teaches(g(x), x))$ (Distribute $\vee$ over $\wedge$)

(v) CNF($\exists x.\forall y.\forall z.(person(x) \wedge ((likes(x, y) \wedge yneqz) \rightarrow \neg likes(x, z))))$)
$\equiv \exists x.\forall y.\forall z.(person(x) \wedge (\neg(likes(x, y) \wedge y \neq z) \vee \neg likes(x, z)))$ (Remove $\rightarrow$)
$\equiv \exists x.\forall y.\forall z.(person(x) \wedge (\neg likes(x, y) \vee y = z \vee \neg likes(x, z)))$ (De Morgan)
$\equiv \forall y.\forall z.(person(x) \wedge (\neg likes(c, y) \vee y = z \vee \neg likes(c, z)))$ (Skolemisation—c is a constant)
$\equiv person(c) \wedge (\neg likes(c, y) \vee y = z \vee \neg likes(c, z))$ (Drop $\forall$)

4. (i) CNF($\forall x.(P(x) \rightarrow Q(x))$)
$\equiv \forall x.(\neg P(x) \vee Q(x))$ (Remove $\rightarrow$)
$\equiv \neg P(x) \vee Q(x)$ (Drop $\forall$)

CNF($\neg \forall x.(\neg Q(y) \rightarrow \neg P(y))$)
$\equiv \neg \forall x.(\neg\neg Q(y) \vee \neg P(y))$ (Remove $\rightarrow$)
$\equiv \exists x.\neg(\neg\neg Q(y) \vee \neg P(y))$ (De Morgan)
$\equiv \exists x.\neg(Q(y) \vee \neg P(y))$ (Double Negation)
$\equiv \exists x.(\neg Q(y) \wedge \neg\neg P(y))$ (De Morgan)
$\equiv \exists x.(\neg Q(y) \wedge P(y))$ (Double Negation)
$\equiv \neg Q(c) \wedge P(c)$ (Skolemisation)

Proof:

1.	$\neg P(x) \vee Q(x)$	(Hypothesis)
2.	$\neg Q(c)$	(Negated Conclusion)
3.	$P(c)$	(Negated Conclusion)
4.	$\neg P(c) \vee Q(c)$	(1. $\{x/c\}$)
5.	$\neg P(c)$	2, 4 Resolution
6.	$\square$	3, 5 Resloution

(ii) (Works exactly as in (i).)

CNF($\forall x.(P(x) \rightarrow Q(x))$)
$\equiv \forall x.(\neg P(x) \vee Q(x))$ (Remove $\rightarrow$)
$\equiv \neg P(x) \vee Q(x)$ (Drop $\forall$)

CNF($\neg \forall x.(\neg Q(x) \rightarrow \neg P(x))$)
$\equiv \neg \forall x.(\neg\neg Q(x) \vee \neg P(x))$ (Remove $\rightarrow$)
$\equiv \neg \forall x.(Q(x) \vee \neg P(x))$ (Double Negation)
$\equiv \exists x.\neg(Q(x) \vee \neg P(x))$ (De Morgan)
$\equiv \exists x.(\neg Q(x) \wedge \neg\neg P(x))$ (De Morgan)
$\equiv \exists x.(\neg Q(x) \wedge P(x))$ (Double Negation)
$\equiv \neg Q(c) \wedge \neg P(c)$ (Skolemisation)

Proof:

1.	$\neg P(x) \vee Q(x)$	(Hypothesis)
2.	$\neg Q(c)$	(Negated Conclusion)
3.	$P(c)$	(Negated Conclusion)
4.	$\neg P(c) \vee Q(c)$	(1. $\{x/c\}$)
5.	$\neg P(c)$	2, 4 Resolution
6.	$\Box$	3, 5 Resloution

(iii) CNF($\forall x.(P(x) \to Q(x))$)
$\equiv \forall x.(\neg P(x) \vee Q(x))$ (Remove $\to$)
$\equiv \neg P(x) \vee Q(x)$ (Drop $\forall$)

CNF($P(a)$)
$\equiv P(a)$

CNF($\neg Q(a)$)
$\equiv \neg Q(a)$

Proof:

1.	$\neg P(x) \vee Q(x)$	(Hypothesis)
2.	$P(a)$	(Hypothesis)
3.	$\neg Q(a)$	(Negated Conclusion)
4.	$\neg P(a) \vee Q(a)$	(1. $\{x/a\}$)
5.	$\neg Q(a)$	2, 4 Resolution
6.	$\Box$	3, 5 Resolution

(iv) CNF($\forall x.(P(x) \to Q(x))$)
$\equiv \forall x.(\neg P(x) \vee Q(x))$ (Remove $\to$)
$\equiv \neg P(x) \vee Q(x)$ (Drop $\forall$)

CNF($\exists x.P(x)$)
$\equiv P(a)$ (Skolemisation)

CNF($\neg \exists x.Q(x)$)
$\equiv \forall x.\neg Q(x)$ (De Morgan)
$\equiv \neg Q(x)$ (Drop $\forall$)

Proof:

1.	$\neg P(x) \vee Q(x)$	(Hypothesis)
2.	$P(a)$	(Hypothesis)
3.	$\neg Q(y)$	(Negated Conclusion)
4.	$\neg P(a) \vee Q(a)$	(1. $\{x/a\}$)
5.	$Q(a)$	2, 4 Resolution
6.	$\neg Q(a)$ (3. $\{y/a\}$)	
7.	$\Box$	5, 6 Resolution

(v) CNF($\forall x.(P(x) \to Q(x))$)
$\equiv \forall x.(\neg P(x) \vee Q(x))$ (Remove $\to$)
$\equiv \neg P(x) \vee Q(x)$ (Drop $\forall$)

CNF($\forall x.(Q(x) \to R(x))$)

$\equiv \forall x.(\neg Q(x) \vee R(x))$ (Remove $\rightarrow$)
$\equiv \neg Q(x) \vee R(x)$ (Drop $\forall$)

CNF($\neg\forall x.(P(x) \rightarrow R(x))$)
$\equiv \neg\forall x.(\neg P(x) \vee R(x))$ (Remove $\rightarrow$)
$\equiv \exists x.(\neg(\neg P(x) \vee R(x))$ (De Morgan)
$\equiv \exists x.(\neg\neg P(x) \wedge \neg R(x))$ (De Morgan)
$\equiv \exists x.(P(x) \wedge \neg R(x))$ (Double Negation)
$\equiv P(c) \wedge \neg R(c)$ (Skolemisation)

Proof:

1.	$\neg P(x) \vee Q(x)$	(Hypothesis)
2.	$\neg Q(y) \vee R(y)$	(Hypothesis)
3.	$P(c)$	(Negated Conclusion)
4.	$\neg R(c)$	(Negated Conclusion)
5.	$\neg P(c) \vee Q(c)$	(1. $\{x/c\}$)
6.	$\neg Q(c) \vee R(c)$	(2. $\{y/c\}$)
7.	$\neg P(c) \vee R(c)$	5, 6 Resolution
8.	$R(c)$	3, 7 Resolution
9.	$\square$	4, 8 Resolution

5. (i) (A) $\exists x.\forall y.(cs(x) \wedge os(y) \wedge likes(x,y))$
(B) $os(Linux)$
(C) $\exists z.(cs(z) \wedge os(Linux) \wedge likes(z, Linux))$

(ii) (A) CNF($\exists x.\forall y.(cs(x) \wedge os(y) \wedge likes(x,y))$)
$\equiv \forall y.(cs(a) \wedge os(y) \wedge likes(a,y))$ (Skolemisation)
$\equiv cs(a) \wedge os(y) \wedge likes(a,y)$ (Drop $\forall$)
(B) CNF($os(Linux)$)
$\equiv os(Linux)$
(C) CNF($\neg\exists z.(cs(z) \wedge os(Linux) \wedge likes(z, Linux))$)
$\equiv \forall z.\neg(cs(z) \wedge os(Linux) \wedge likes(z, Linux))$ (De Morgan Laws)
$\equiv \forall z.(\neg cs(z) \vee \neg os(Linux) \vee \neg likes(z, Linux))$ (De Morgan Laws)
$\equiv \neg cs(z) \vee \neg os(Linux) \vee \neg likes(z, Linux)$ (Drop $\forall$)

(iii)

1.	$cs(a)$	(Hypothesis A)
2.	$os(w)$	(Hypothesis A)
3.	$likes(a, x)$	(Hypothesis A)
4.	$os(Linux)$	(Hypothesis B)
5.	$\neg cs(z) \lor \neg os(Linux) \lor \neg likes(z, Linux)$	(Negated Conclusion)
6.	$\neg cs(a) \lor \neg os(Linux) \lor \neg likes(a, Linux)$	(5. $\{z/a\}$)
7.	$\neg os(Linux) \lor \neg likes(a, Linux)$	(1, 6 Resolution)
8.	$likes(a, Linux)$	(3. $\{x/Linux\}$)
9.	$\neg os(Linux)$	(7, 8 Resolution)
10.	$os(Linux)$	(3. {w/Linux})
11.	$\Box$	(9, 10 Resolution)

(iv) Yes. A, B, $\neg C$ in (i) are Horn clauses so there must be an SLD resolution of the empty clause if there is a resolution of the empty clause. In fact, the resolution in (ii) is an SLD resolution of the empty clause.

(v) $A, B \vdash C$

6. (i) (A) $\forall s.((r(s) \lor p(s)) \land l(s) \to a(s))$

(B) $\exists s.\neg a(s)$

(C) $\exists s.(l(s) \land \neg r(s) \land \neg p(s))$

(ii) (A) CNF($\forall s.((r(s) \lor p(s)) \land l(s) \to a(s))$)

$\equiv \forall s.(\neg((r(s) \lor p(s)) \land l(s)) \lor a(s))$ (Remove $\to$)

$\equiv \forall s.(\neg(r(s) \lor p(s)) \lor \neg l(s) \lor a(s))$ (De Morgan)

$\equiv \forall s.((\neg r(s) \land \neg p(s)) \lor \neg l(s) \lor a(s))$ (De Morgan)

$\equiv (\neg r(s) \land \neg p(s)) \lor \neg l(s) \lor a(s)$ (Drop $\forall$)

(B) CNF($\exists s.\neg a(s)$)

$\equiv \neg a(c)$ (Skolemisation)

(C) CNF($\neg\exists s.(l(s) \land \neg r(s) \land \neg p(s))$)

$\equiv \forall s.\neg(l(s) \land \neg r(s) \land \neg p(s))$ (De Morgan)

$\equiv \forall s.(\neg l(s) \lor r(s) \lor p(s))$ (De Morgan and double negation)

$\equiv \neg l(s) \lor r(s) \lor p(s)$ (Drop $\forall$)

(iii)

1.	$\neg r(s) \land \neg p(s)$	(Hypothesis A)
2.	$\neg l(s)$	(Hypothesis A)
3.	$a(s)$	(Hypothesis A)
4.	$\neg a(c)$	(Hypothesis B)
5.	$\neg l(s) \lor r(s) \lor p(s)$	(Negated Conclusion)
6.	$\neg l(s)$	(1,5 Resolution)

(iv) No. The empty clause cannot be derived by resolution.

(v) $A, B \nvdash C$