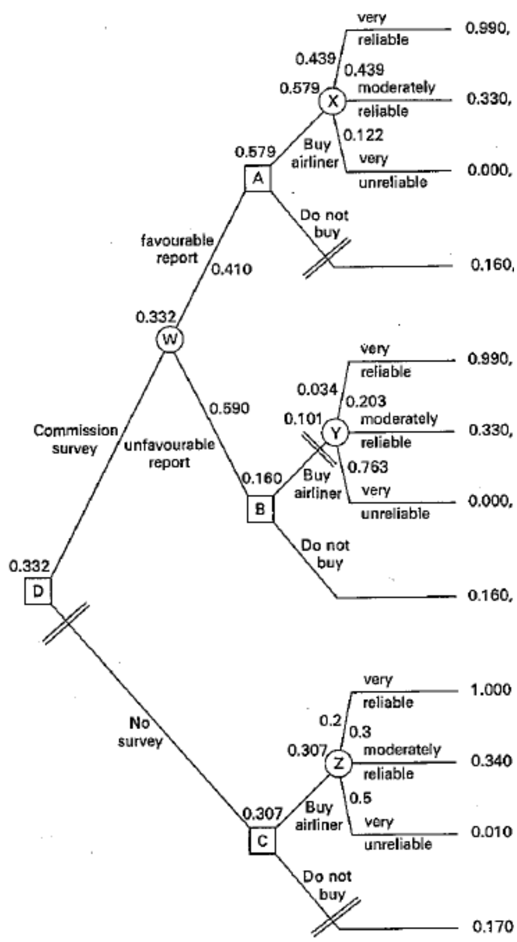
GSOE9210 Engineering Decisions

Problem Set 07

1. Consider the airliner problem discussed in lectures:



The decision-maker's epistemic state (*i.e.*, *prior* probabilities), based on general industry information, and preferences (utilities) for buying an aircraft, based on its operational reliability, are given below.

	Reliability		
	vR	mR	uR
Probability	0.2	0.3	0.5
Utility	1.0	0.34	0.01

The values at decision points A and B in the diagram above are based on the original consulting firm's (call it F_1) accuracy:

	... given:		
Probability of:	vR	mR	uR
f	0.9	0.6	0.1
u	0.1	0.4	0.9

Consider a second consulting firm (F_2) which charges the same \$10,000 fee, but for which:

	... given:		
Probability of:	vR	mR	uR
f	0.6	0.5	0.4
u	0.4	0.5	0.6

(a) Verify that subtracting a fixed value from the utility of each outcome (*e.g.*, the cost of the report) of the sub-tree with root A results in the same reduction in the value of node A.

(b) Would you expect the value of the information provided by F_2 to be better or worse than that of F_1?

(c) Calculate the probabilities of the airliner's reliabilities (*i.e.*, $P(vR|f)$, *etc.*) based on a favourable assessment by F_2; *i.e.*, the updated probabilities for the branches at node X.

(d) Repeat the above for node Y.

(e) What are the utilities of buying the airliner if the report is favourable and unfavourable (nodes X and Y) respectively?

(f) What would be the utility of commissioning the report if it turns out to be favourable (*i.e.*, the value at node A) and unfavourable (*i.e.*, the value at node B) respectively?

(g) For firm F_2, determine the updated likelihoods of the report being favourable (f) and unfavourable (u) (*i.e.*, the probabilities associated with the branches at node W).

(h) What is the value of commissioning F_2's report (*i.e.*, the utility at node W)?

(i) Would it be worthwhile paying F_2's \$10,000 fee?

(j) Suppose a third company, F_3, regularly gave incorrect advice:

	... given:		
Probability of:	vR	mR	uR
f	0.1	0.5	0.8
u	0.9	0.5	0.2

How valuable would you expect their information to be?

2. Consider five possible prizes/outcomes, $x_1, \ldots, x_5$, listed by a rational agent in non-increasing order of preference (*i.e.*, $x_1 \succsim x_2 \succsim \cdots \succsim x_5$). Further, assume that when interviewed further the agent is unable to give precise preferences but specifies the following:

A $[0.9 : x_1|0.1 : x_5] \succ x_2 \succ [0.8 : x_1|0.2 : x_5]$

B $[0.42 : x_1|0.2 : x_4|0.38 : x_5] \succ [0.3 : x_1|0.6 : x_4|0.1 : x_5] \succ [0.38 : x_1|0.2 : x_4|0.42 : x_5]$

C $[0.7 : x_1|0.3 : x_5] \succ x_3 \succ [0.5 : x_2|0.5 : x_4]$

Given the uncertainty in the agent's utility estimates:

(a) Find the range of utility values for each of $x_1, \ldots, x_5$. You may assume utilities are in the range $[0, 1]$.

(b) Determine the agent's preference relation (*i.e.*, $\succ$, $\sim$, or indeterminate[1]) between the two lotteries: $[0.5 : x_3|0.5 : x_4]$ and $[0.5 : x_2|0.5 : x_3]$.

(c) Determine the agent's preference relation between the lottery $[0.3 : x_1|0.1 : x_2|0.5 : x_3|0.1 : x_4]$ and the outcome x_3[2]

(d) Determine the agent's preference relation between the two lotteries $[0.1 : x_2|0.6 : x_3|0.3 : x_4]$ and $[0.1 : x_2|0.7 : x_3|0.2 : x_4]$

(e) Determine the agent's preference relation between the two lotteries $[0.5 : x_1|0.5 : x_4]$ and $[0.2 : x_1|0.6 : x_3|0.2 : x_5]$

[1] Because the utilities of the prizes are not precisely determined, it may be that for some utility values one lottery is preferred to another and for other values the opposite is the case. In this case the preference relation would be *indeterminate*.

[2] You can think of the latter as a certain lottery; one which always results in x_3.