Computer Vision Week 4

COMP9517
Locality Sensitive Hashing

- Take random projections of data
- Quantize each projection with few bits
Finding a parametric transformation

- Similarity (translation, scale, rotation)
- Affine
- Projective (homography)

Slide: S. Lazebnik
Panograph using only rotation and translation
Fitting an affine transformation

• Assume we know the correspondences, how do we get the transformation?

\[
\begin{bmatrix}
  x' \\
  y'
\end{bmatrix} = \begin{bmatrix}
  m_1 & m_2 \\
  m_3 & m_4
\end{bmatrix} \begin{bmatrix}
  x_i \\
  y_i
\end{bmatrix} + \begin{bmatrix}
  t_1 \\
  t_2
\end{bmatrix}
\]

\[
\begin{bmatrix}
  x_i & y_i & 0 & 0 & 1 & 0 \\
  0 & 0 & x_i & y_i & 0 & 1 \\
  \vdots \\
  m_1 \\
  m_2 \\
  m_3 \\
  m_4 \\
  t_1 \\
  t_2
\end{bmatrix} \begin{bmatrix}
  m_1 \\
  m_2 \\
  m_3 \\
  m_4 \\
  t_1 \\
  t_2
\end{bmatrix} = \begin{bmatrix}
  x' \\
  y'
\end{bmatrix}
\]

Slide: S. Lazebnik
Fitting an affine transformation

\[
\begin{bmatrix}
\vdots \\
x_i & y_i & 0 & 0 & 1 & 0 \\
0 & 0 & x_i & y_i & 0 & 1 \\
\vdots \\
\end{bmatrix}
\begin{bmatrix}
m_1 \\
m_2 \\
m_3 \\
m_4 \\
t_1 \\
t_2 \\
\end{bmatrix}
= \begin{bmatrix}
\vdots \\
x'_i \\
y'_i \\
\vdots \\
\end{bmatrix}
\]

- Linear system with six unknowns
- Each match gives us two linearly independent equations: need at least three to solve for the transformation parameters
- Can solve \( Ax=b \) using least squares:
  \[ x = (A^T A)^{-1} A^T b; \quad \text{or } x = A \backslash b; \text{ in Matlab} \]
Dealing with outliers

• The set of putative matches still contains a very high percentage of outliers

• How do we fit a geometric transformation to a small subset of all possible matches?

• Possible strategies:
  – RANSAC
  – Robust alternatives to least squares
Least squares: Robustness to noise

- Least squares fit to the red points:
Least squares: Robustness to noise

• Least squares fit with an outlier:

Problem: squared error heavily penalizes outliers
Robust estimators

- General approach: minimize \( \sum_i \rho(r_i(x_i, \theta); \sigma) \)

\( r_i(x_i, \theta) \) – residual of ith point w.r.t. model parameters \( \theta \)
\( \rho \) – robust function with scale parameter \( \sigma \)

The robust function \( \rho \) behaves like squared distance for small values of the residual \( u \) but saturates for larger values of \( u \)
Choosing the scale: Just right

The effect of the outlier is eliminated
Choosing the scale: Too small

The error value is almost the same for every point and the fit is very poor.
Choosing the scale: Too large

Behaves much the same as least squares
Robust estimation: Notes

- Robust fitting is a nonlinear optimization problem that must be solved iteratively
- Use IRLS algorithm
- Least squares solution can be used for initialization