GSOE9210 Engineering Decisions

Problem Set 05

1. Consider the river problem described in lectures:

	p	$1-p$	
	f	$\overline{f}$	V_B
A	4	0	$4p$
B	3	1	$2p+1$

   (a) For $p = \frac{3}{4}$, what is the slope of the *Bayes* indifference line through A?

   (b) Draw the *Bayes* indifference curves for $p = \frac{1}{4}$ and $\frac{3}{4}$ through A and B.

   (c) Draw the *Bayes* indifference curve for which an agent would be indifferent between A and B, respectively. What is the slope of the line?

   (d) For which probability (*i.e.*, value of p) would an agent be indifferent between A and B under the *Bayes* decision rule?

   (e) What is the *Bayes* value associated with the indifference curve through A and B?

   (f) For which values of p would an agent prefer A to B?

2. Repeat the above exercises for regret. What can you infer about the *Bayes* decision rule when applied to the original values versus regrets?

3. Consider the generic two-strategy problem below:

	p	$1-p$
	s_1	s_2
A	a_1	a_2
B	b_1	b_2

   Assume neither strategy dominates the other.

   (a) Prove that an agent will be indifferent between A and B under *Bayes* when:

$$p = \frac{\Delta y}{\Delta x + \Delta y}$$

   where

$$\Delta y = |a_2 - b_2|$$
$$\Delta x = |a_1 - b_1|$$

(b) Prove that:

$$p = \frac{m}{m-1}$$

where $m = \frac{\Delta y}{\Delta x}$ is the slope of the line joining A and B in the Cartesian plane.

4. Consider the decision table below, with $P(s_1) = p$:

	p	$1-p$
	s_1	s_2
A	5	3
B	4	1
C	2	5

(a) For which value of p would the agent be indifferent between A and C?

(b) Plot the *Bayes* values for the strategies as p varies from 0 to 1.

(c) For which values of p are A, B, and C preferred, respectively, under the *Bayes* decision rule?

5. Each day, a drinks vendor must purchase stock of several types of drink to sell in her shop. The types of drink which may be stocked are: a) hot chocolate; b) iced tea; c) lemonade; d) orange juice.

   She knows, from past experience, that on warm days she'll make sales totalling \$10 on hot chocolate, \$40 on iced tea, \$30 on lemonade, and \$40 on orange juice. On cool days, however, her sales total \$30 on hot chocolate, \$0 on iced tea, \$20 on lemonade, and \$10 on orange juice.

(a) Produce a decision table for this problem.

(b) What proportion of drinks should she stock to maximise her guaranteed (*i.e.*, minimum) sales total regardless of the temperature?

(c) Find the *Bayes* strategies for $p = 0, \frac{1}{4}, \frac{1}{2}, \frac{3}{4}, 1$.

(d) What is the least favourable probability distribution on warm and cool (not warm) days?

(e) Repeat the above analysis for the *miniMax Regret* rule.

(f) Define the admissibility frontier for this problem.

6. Show that a strategy is admissible iff it is a *Bayes* strategy for some probability distribution.

7. Show that a *Maximin* strategy is always a *Bayes* strategy for some probability distribution.

8. Prove that for any two actions A and B, if A weakly dominates B, and all state probabilities are non-zero, then the *Bayes* decision rule will *strictly* prefer A over B.