COMP9334
Capacity Planning for Computer Systems and Networks

Week 1: Revision problem set

Question 1

- An important part of performance analysis is to model the workload. In this question, you will look at a very simple model and we will generalise it to a very well known model in performance analysis in the lecture in Week 2.
- Consider a user who may send HTTP requests to a web server. In the time interval $[k\delta, (k+1)\delta)$ where k is a non-negative integer, there is a probability of p that this user will send an HTTP request to a web server and there is a probability of $(1-p)$ that this user will not send. Assuming that the probability the user sends (or not send) in each time interval is independent. Assuming that the current time is 10δ, what is the probability that this user will not send an HTTP request to the web server before 30δ ?

Question 2

- This is a revision question on probability distribution which you should be able to solve if you have the pre-requisites.
- Consider a continuous probability distribution with sample space is $[1,\infty)$ and probability density function
  - $f(x) = a / x^3$ for $x \geq 1$
- What is the value of *a* in order that f(x) be a valid probability density function?
- Given this probability density function, what is the probability that a number drawn from this distribution has a value greater than 10?

Question 1 - Answers

Prob (the user will not send before 30δ)

= Prob (the user will not send in [10δ,11δ)) x

Prob (the user will not send in [11δ,12δ)) x ….

Prob (the user will not send in [29δ,30δ))

(note: the probability to send is independent for each time)

= $(1-p)^{20}$

Question 2 - Answers

- In order that the probability density function be valid, the probability that the number is drawn between [1,∞) is 1.

$$\int_1^\infty \frac{a}{x^3} = 1 \Rightarrow \left[\frac{ax^{-2}}{-2}\right]_1^\infty = 1 \Rightarrow \frac{a}{2} = 1 \Rightarrow a = 2$$

- Probability that a number drawn is greater than 10 =

$$\int_{10}^\infty \frac{2}{x^3} = \left[\frac{2x^{-2}}{-2}\right]_{10}^\infty = 0.01$$

- Note: The probability distribution that you've worked with is called a Pareto distribution. It has what is known as a heavy tail properties. This probability distribution appears very often in modern computer performance analysis.