# Exercise Sheet 2
# COMP6741: Parameterized and Exact Computation

2016, Semester 2

1. Prove the following generalization of Lemma 3 [Lawler '76]: For any graph $G$ on $n$ vertices, if $G$ has a $k$-coloring, then $G$ has a $k$-coloring where one color class is a maximal independent set in $G$ of size at least $n/k$.

2. In the MEETING MOST DEADLINES problem, we are given $n$ tasks $t_1, \ldots, t_n$, and each task $t_i$ has a length $\ell_i$, a due date $d_i$, and a penalty $p_i$ which applies when the due date of task $t_i$ is not met. The problem asks to assign a start date $s_i \geq 0$ to each task $t_i$ so that the executions of no two tasks overlap, and the sum of the penalties of those tasks that are not finished by the due date is minimized.

| MEETING MOST DEADLINES | |
|---|---|
| Input: | A set $T = \{t_1, \ldots, t_n\}$ of $n$ tasks, where each task $t_i$ is a triple $(\ell_i, d_i, p_i)$ of three non-negative integers. |
| Output: | A schedule, assigning a start date $s_i \in \mathbb{N}_0$ to each task $t_i \in T$ such that $$\sum_{i \in \{1,\ldots,n\} \,:\, s_i + \ell_i > d_i} p_i$$ is minimized, subject to the constraint that for every $i, j \in \{1, \ldots, n\}$ with $i \neq j$ we have that $s_i \notin \{s_j, s_j + 1, \ldots, s_j + \ell_j - 1\}$. |

(a) Show that the MEETING MOST DEADLINES problem can be solved in $O^*(n!)$ time by reformulating it as a permutation problem.

(b) Design an algorithm solving the MEETING MOST DEADLINES problem in $O^*(2^n)$ time.