Exercise sheet 8a – Solutions and Hints
COMP6741: Parameterized and Exact Computation
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Exercise 1. Design a randomized FPT algorithm for 1-Regular Deletion with running time $O^*(6^k)$

**1-Regular Deletion**
- **Input:** Graph $G = (V, E)$, integer $k$
- **Parameter:** $k$
- **Question:** Does there exist $X \subseteq V$ with $|X| \leq k$ such that $G - X$ is 1-regular?

**Solution sketch.**
- If there is a vertex with degree 0, then remove it and decrease $k$ by 1.
- If there is an isolated edge $uv$, then remove $u$ and $v$.
- If there is an isolated $P_3$, $(u, v, w)$, then remove $u, v, w$, and decrease $k$ by 1.
- The graph now has average degree at least 1.5. A 1-regular deletion set $X$ is incident to at least $|E|/3$ edges.
- Choose an edge uniformly at random and then an endpoint of the chosen edge uniformly at random for a $1/6$ probability of selecting a vertex in $X$.

Exercise 2. Design a randomized FPT algorithm for Triangle Packing.

**Triangle Packing**
- **Input:** Graph $G$, integer $k$
- **Parameter:** $k$
- **Question:** Does $G$ have $k$ vertex-disjoint triangles?

**Solution sketch.**

**By Color Coding**
- Suppose there is a subset $X \subseteq V$ of size $3k$ that induces $k$ vertex-disjoint triangles. By Lemma 9, a random $3k$-coloring $\chi$ of the vertices colors $X$ with pairwise distinct colors with probability at least $e^{-3k}$.
- For a graph $G$ and coloring $\chi : V(G) \rightarrow [3k]$, in a similar manner to Lemma 10 we design an algorithm that checks whether $G$ contains a triangle packing on $3k$ vertices such that all vertices are pairwise distinctly colored:
  - Enumerate all possible ways of partitioning $3k$ colors into $k$ bags of exactly 3 colors each. There are exactly $\frac{(3k)!}{(3!)^k \cdot k!}$ such ways.
  - For each bag, we need to check whether there is a triangle in $G$ with colors from this bag.
  - For a bag, let these colors be $i, j, k$ and consider the vertices with these colors, $V_i, V_j, V_k$. We now check if there exists a triangle using one vertex from each of the sets $V_i, V_j, V_k$. This can be done in time $n^3$. Repeating this for all $k$ bags only requires $k \cdot n^3$ time.
- Final running time: $O^*(e^{3k} \cdot \frac{(3k)!}{(3!)^k \cdot k!})$. 
