Exercise sheet 8a – Solutions and Hints

COMP6741: Parameterized and Exact Computation

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Exercise 1. Design a randomized FPT algorithm for 1-Regular Deletion with running time $O^*(6^k)$

1-Regular Deletion	
Input:	Graph $G = (V, E)$, integer k
Parameter:	k
Question:	Does there exist $X \subseteq V$ with $|X| \leq k$ such that $G - X$ is 1-regular?

Solution sketch.

- If there is a vertex with degree 0, then remove it and decrease k by 1.
- If there is an isolated edge uv, then remove u and v.
- If there is an isolated P_3, (u, v, w), then remove u, v, and w, and decrease k by 1.
- The graph now has average degree at least 1.5. A 1-regular deletion set X is incident to at least $\frac{|E|}{3}$ edges.
- Choose an edge uniformly at random and then an endpoint of the chosen edge uniformly at random for a $\frac{1}{6}$ probability of selecting a vertex in X.

Exercise 2. Design a randomized FPT algorithm for Triangle Packing.

Triangle Packing	
Input:	Graph G, integer k
Parameter:	k
Question:	Does G have k vertex-disjoint triangles?

Solution sketch.

By Color Coding

- Suppose there is a subset $X \subseteq V$ of size $3k$ that induces k vertex-disjoint triangles. By Lemma 9, a random $3k$-coloring χ of the vertices colors X with pairwise distinct colors with probability at least e^{-3k}.
- For a graph G and coloring $\chi : V(G) \to [3k]$, in a similar manner to Lemma 10 we design an algorithm that checks whether G contains a triangle packing on $3k$ vertices such that all vertices are pairwise distinctly colored:
  - Enumerate all possible ways of partitioning $3k$ colors into k bags of exactly 3 colors each. There are exactly $\frac{(3k)!}{(3!)^k \cdot k!}$ such ways.
  - For each bag, we need to check whether there is a triangle in G with colors from this bag.
  - For a bag, let these colors be i, j, k and consider the vertices with these colors, V_i, V_j, V_k. We now check if there exists a triangle using one vertex from each of the sets V_i, V_j, V_k. This can be done in time n^3. Repeating this for all k bags only requires $k \cdot n^3$ time.
- Final running time: $O^*(e^{3k} \cdot \frac{(3k)!}{(3!)^k \cdot k!})$.