# COMP4418
## Knowledge Representation and Reasoning

# Week 3 – Practical Reasoning

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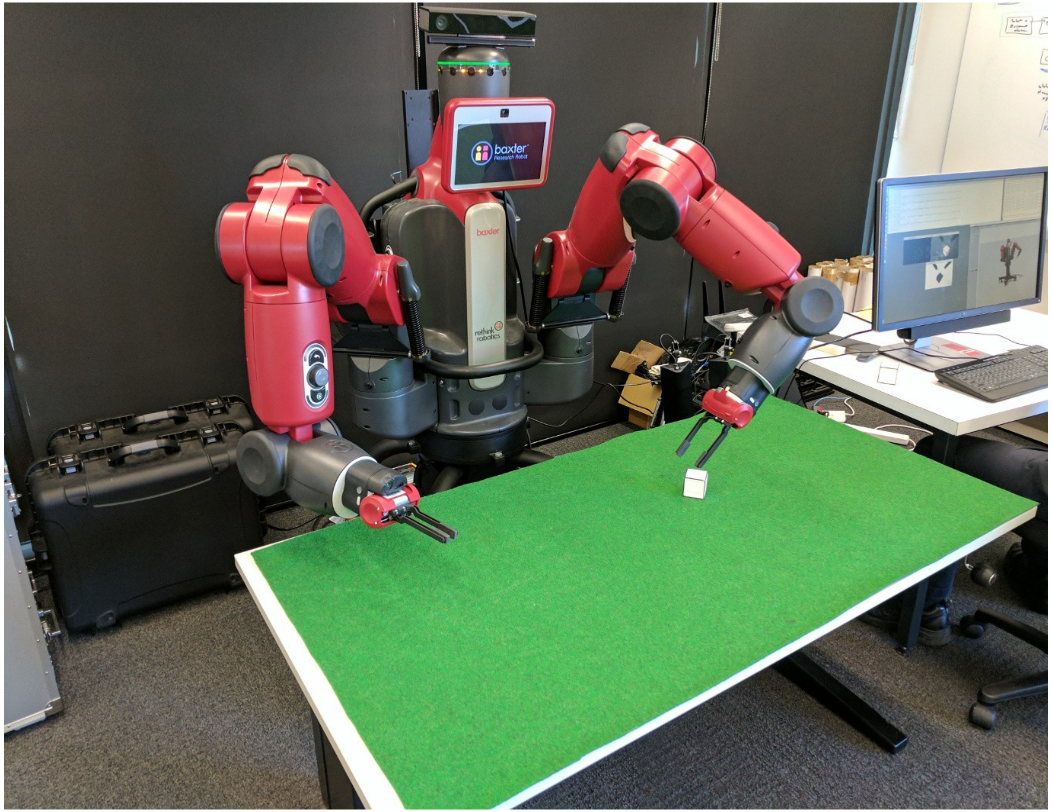
# Practical Reasoning - My Interests

- Cognitive Robotics.
- Connect high level cognition with low-level sensing/actuators.
- Logical reasoning to make robot behave intelligently.
- Baxter Blocksworld video…

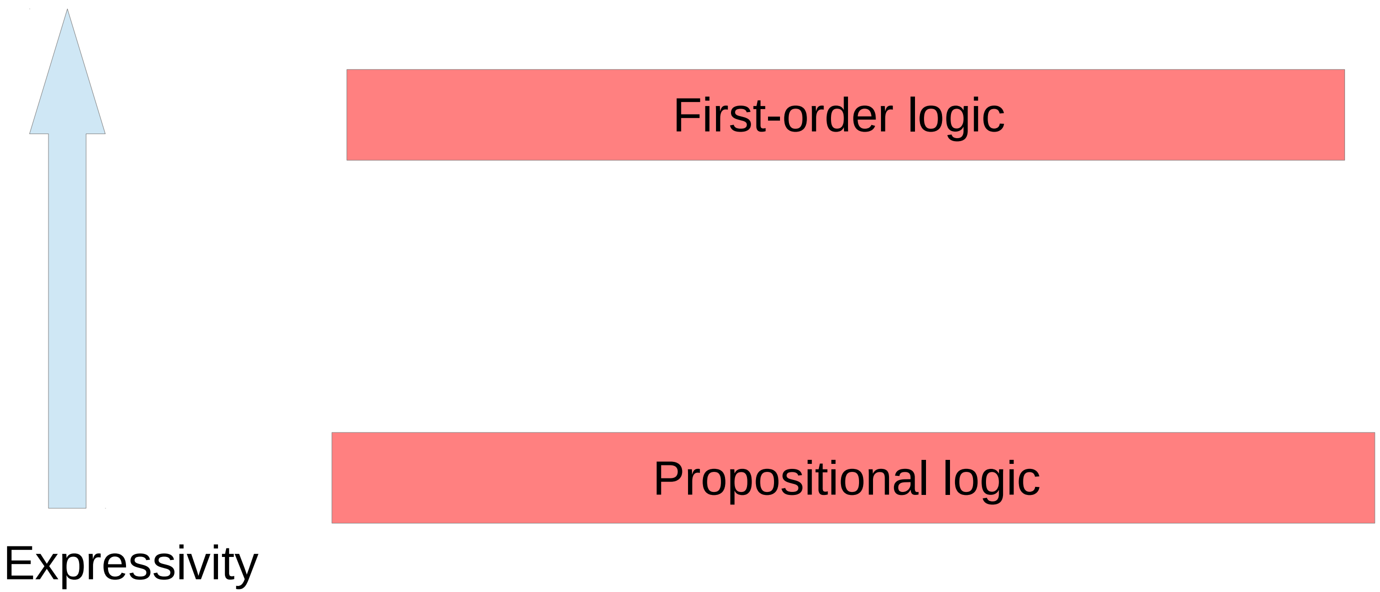


# Recap of Weeks 1 & 2

- Week 1: Propositional logic
  - Simple propositions: “Socrates is bald”
  - Semantics: meaning decided using truth tables
  - Syntax: provability decided using inference rules – resolution for CNF
  - But… limited expressivity
- Week 2: First-order logic
  - Able to capture properties of objects and relationships between objects
  - Semantics: meaning decided using interpretations
  - Syntax: provability using inference rules - resolution + unification for CNF
  - highly expressive but… undecidable.

# A Brief Overview of KRR Formalisms

# Many Formalisms in KRR



# Many Formalisms in KRR

First-order logic – Satisfiability is undecidable

*actually semi-decidable, but distinction is not important for this course.

Propositional logic – Satisfiablity is NP-complete

Expressivity

Computational Complexity

# Many Formalisms in KRR

First-order logic – Satisfiability is undecidable

Propositional logic – Satisfiablity is NP-complete

Expressivity

Computational Complexity

Many important problems:

- Scheduling
- Timetabling
- Vehicle routing

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# Many Formalisms in KRR

First-order logic – Satisfiability is undecidable

Propositional logic – Satisfiablity is NP-complete

Propositional fragments

Expressivity

Computational Complexity

When speed is important:

- Databases

# Many Formalisms in KRR

Higher-order logics – some interest

First-order logic – Satisfiability is undecidable

Propositional logic – Satisfiablity is NP-complete

Propositional fragments

Expressivity

Computational Complexity

When speed is important:

- Databases

# Many Formalisms in KRR

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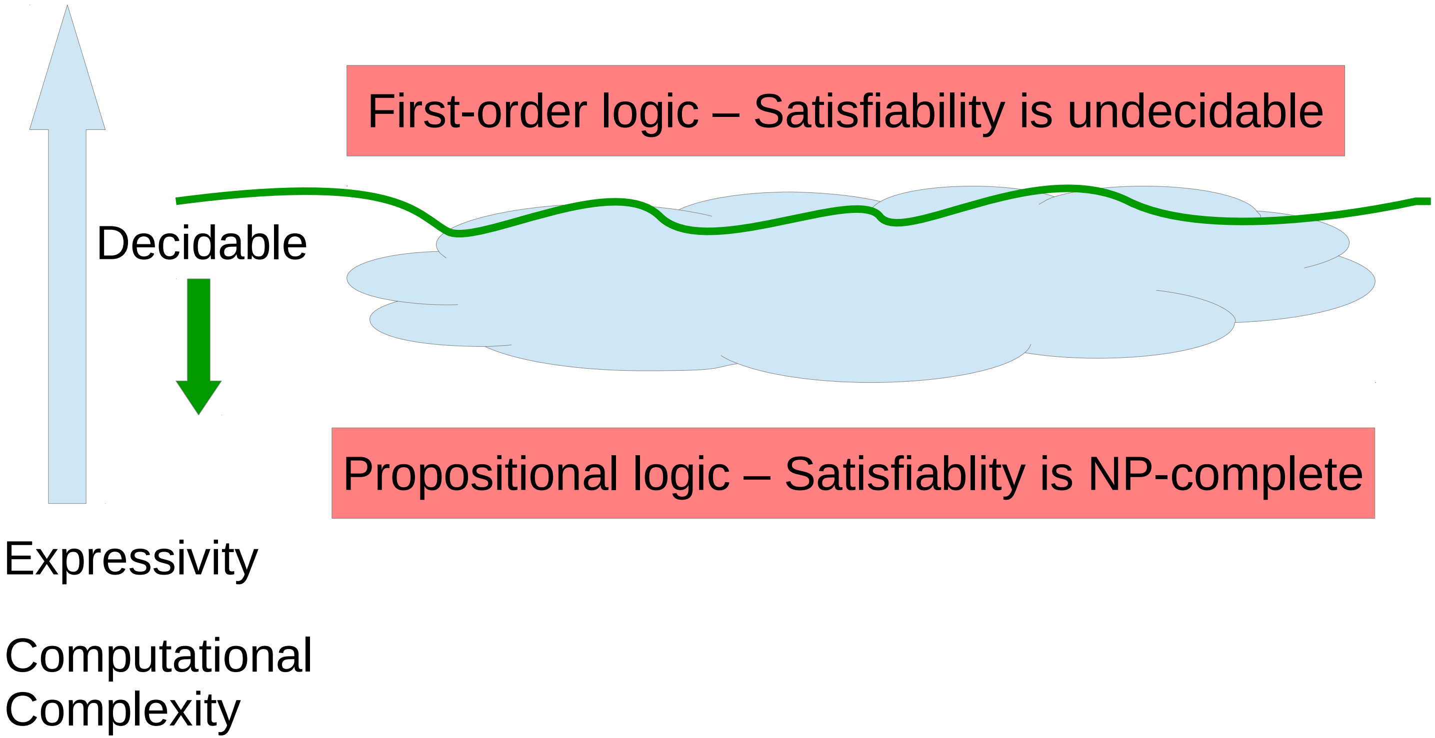
# Many Formalisms in KRR



# Many Formalisms in KRR

First-order logic – Satisfiability is undecidable

Decidable

Description logics, modal logics, Answer Set Programming

Propositional logic – Satisfiablity is NP-complete

Expressivity

Computational Complexity

Many interesting problems:

- Planning
- Reasoning about knowledge

# Horn Clauses

# Clause Recap

From weeks 1 & 2:

- Every formula can be converted to Conjunctive Normal Form (CNF)
- Any CNF can be viewed as a set of clauses
- Entailment checking with resolution is complete (proof by refutation)
- So using sets of clauses provides:
  - Intuitive language for expressing knowledge

    $\neg a, a \vee b$ vs $\neg(a \vee (\neg a \wedge \neg b))$

  - Simple proof procedure that can be implemented

# Reading Clauses as Implication

Clauses can be intuitively interpreted in two ways:

- As disjunction: $\text{rain} \lor \text{sleet}$
- As implication: $\neg\text{child} \lor \neg\text{male} \lor \text{boy}$
  - for syntactic convenience: $\text{child} \land \text{male} \rightarrow \text{boy}$
  - so can be read as: if "child" and "male" then "boy"

To understand why this makes sense go back to the truth tables:

| A | B | $\neg A$ | $\neg A \lor B$ | $A \rightarrow B$ |
|---|---|---|---|---|
| True | True | False | True | True |
| True | False | False | False | False |
| False | True | True | True | True |
| False | False | True | True | True |

# Horn Clauses

- ***Horn clause*** is a clause with at most one positive literal
- A ***positive*** (or ***definite) clause*** has exactly one positive literal

  $\neg\text{child} \vee \neg\text{male} \vee \text{boy}$

- A ***negative clause*** (or ***constraint***) has no positive literals

  $\neg\text{open} \vee \neg\text{closed}$

  – Note, since $\neg\text{open} \vee \neg\text{closed} \equiv \neg\text{open} \vee \neg\text{close} \vee \text{False}$
  – Hence $\text{open} \wedge \text{closed} \rightarrow \text{False}$ ( $\text{open} \wedge \text{closed} \rightarrow \perp$ or $\text{open} \wedge \text{closed} \rightarrow$)
  – Also know as a **goal** when performing refutation proof

- A ***fact*** is a definite clause with no negative literals (i.e., a single positive literal):

  $\text{raining}$

# Resolution with Horn Clauses 1

Two options:

Neg, Pos → Neg

Pos, Pos → Pos

Examples:

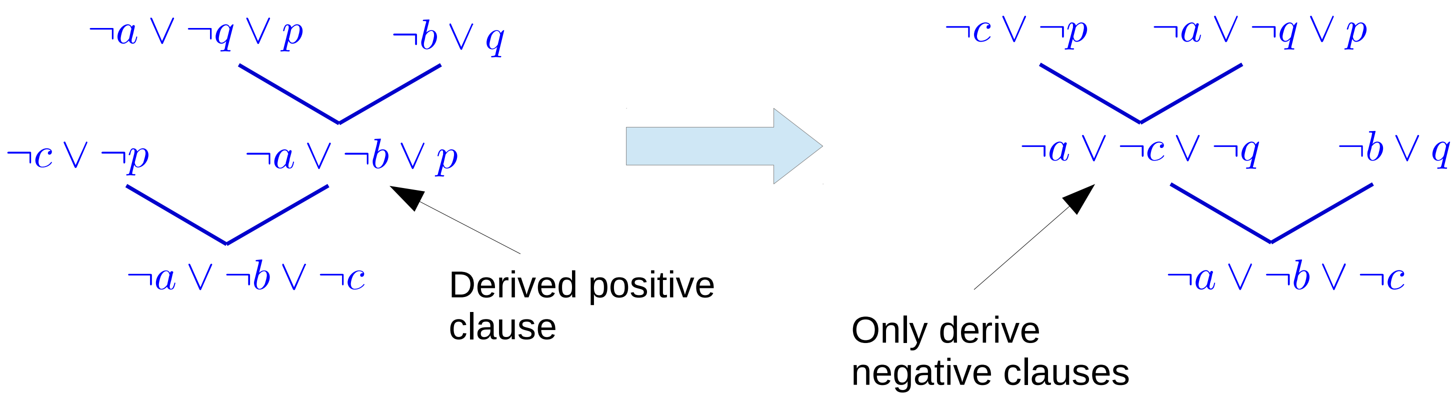
$\neg a \vee \neg b$, $a \vee \neg c$ → $\neg b \vee \neg c$

$\neg a \vee b$, $a \vee \neg c$ → $\neg c \vee b$

# Resolution with Horn Clauses 2

It is possible to rearrange derivations (of negative clauses) so that all new derived clauses are negative clauses:

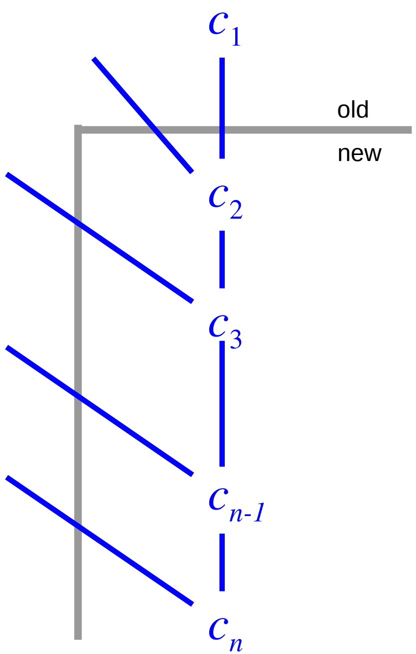
Given clauses: $\neg a \vee \neg q \vee p$ $\quad \neg b \vee q$ $\quad \neg c \vee \neg p$



# SLD Resolution

Can change derivations such that each derived clause is a resolvent of the previous derived (negative) one and some positive clause in the original set of clauses

- Since each derived clause is negative, one parent must be positive (and so from original set) and one negative.
- Continue working backwards until both parents of derived clause are from the original set of clauses
- Eliminate all other clauses not on direct path



# SLD Example

To show that $\mathrm{KB} \models \mathrm{Girl}$ derive a contradiction from $\mathrm{KB} \cup \{\neg\mathrm{Girl}\}$

$$
\begin{aligned}
\mathrm{KB} = \{ \quad & \mathrm{FirstGrade}, \\
& \mathrm{FirstGrade} \rightarrow \mathrm{Child}, \\
& \mathrm{Child} \wedge \mathrm{Male} \rightarrow \mathrm{Boy}, \\
& \mathrm{Kindergarten} \rightarrow \mathrm{Child}, \\
& \mathrm{Child} \wedge \mathrm{Female} \rightarrow \mathrm{Girl}, \\
& \mathrm{Female} \quad \}
\end{aligned}
$$

$\neg\mathrm{Child} \vee \neg\mathrm{Female} \vee \mathrm{Girl}$ $\qquad$ $\neg\mathrm{Girl}$

$\mathrm{Female}$ $\qquad$ $\neg\mathrm{Child} \vee \neg\mathrm{Female}$

$\neg\mathrm{FirstGrade} \vee \mathrm{Child}$ $\qquad$ $\neg\mathrm{Child}$

$\mathrm{FirstGrade}$ $\qquad$ $\neg\mathrm{FirstGrade}$

$\perp$

Note: Horn clauses capture a very intuitive way that we express knowledge.

# SLD Resolution (formal)

An <u>SLD-derivation</u> of a clause $c$ from a set of clauses $S$ is a sequence of clauses $c_1, c_2, \ldots c_n$ such that $c_n = c$, and

1. $c_1 \in S$
2. $c_{i+1}$ is a resolvent of $c_i$ and a clause in $S$

Written as: $S \vdash^{\mathrm{SLD}} c$

**SLD** mean S(elected) literals
L(inear) form
D(efinite) clauses

# In General SLD is incomplete

SLD resolution is not complete for general clauses.

An example: $S = \{\, p \lor q,\ p \lor \neg q,\ \neg p \lor q,\ \neg p \lor \neg q \,\}$

$$\dfrac{\dfrac{p \lor q \qquad p \lor \neg q}{p} \qquad \dfrac{\neg p \lor q \qquad \neg p \lor \neg q}{\neg p}}{\bot}$$

So S is unsatisfiable, that is: $\mathrm{S} \vdash \bot$, but $\mathrm{S} \not\vdash^{\mathrm{SLD}} \bot$

SLD cannot derive the contradition because it needs to eventually perform resolution on the intermediate clauses $p$ and $\neg p$ (or $q$ and $\neg q$)

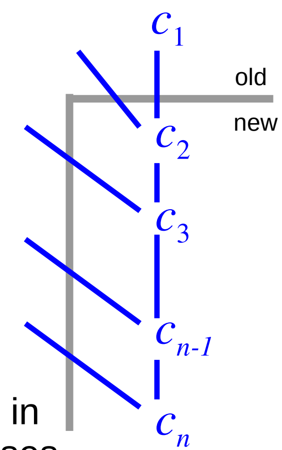
# Completeness of SLD

- But SLD resolution IS complete for Horn clauses.

**Theorem**: If H is a set of Horn clauses then $\mathrm{H} \vdash \bot$ iff $\mathrm{H} \vdash^{\mathrm{SLD}} \bot$

- This is a good result as searching for appropriate clauses to resolve on is simpler for SLD resolution.
- Satisfiability for propositional Horn clauses is P-complete.
- Nothing is for free: **loss of expressivity.**
- Cannot express simple (positive) disjunctions.

$$\text{open} \lor \text{closed}$$




*n* is polynomial in number of clauses

# Back to the KRR Overview

First-order logic – Satisfiability is undecidable

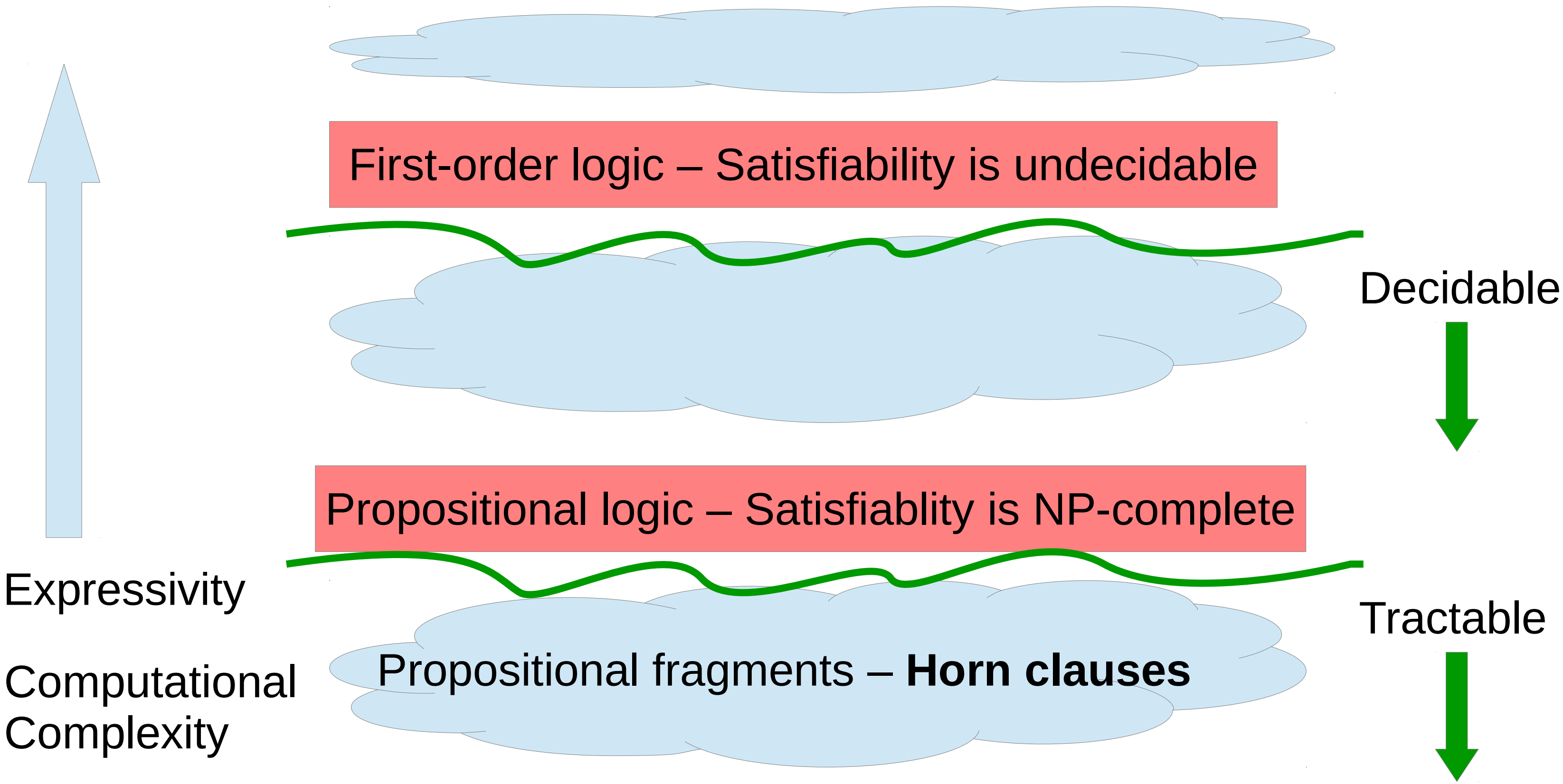
Propositional logic – Satisfiablity is NP-complete

Expressivity

Computational Complexity

Propositional fragments – **Horn clauses**

# Back to the KRR Overview



# First-Order (FO) Clauses

Week 2 recap:

- Conversion to FO CNF is same as propositional case except:
  - Standardise variable names
  - Skolemise (getting rid of existential quantifiers)
  - Drop universal quantifiers
- FO resolution is same as propositional case except:
  - Find substitutions to unify the two clauses

# First-Order (FO) Horn Clauses

- Same as propositional case except in a FO language
- SLD-resolution also same; with addition of unification
- Completeness of FO Horn also holds

**Theorem**: If H is a set of Horn clauses then $\mathrm{H} \vdash \perp$ iff $\mathrm{H} \vdash^{\mathrm{SLD}} \perp$

- But…

# First-Order (FO) Horn Clauses

- FO Horn is undecidable. With Horn SLD resolution we can still generate an infinite sequence of resolvents.

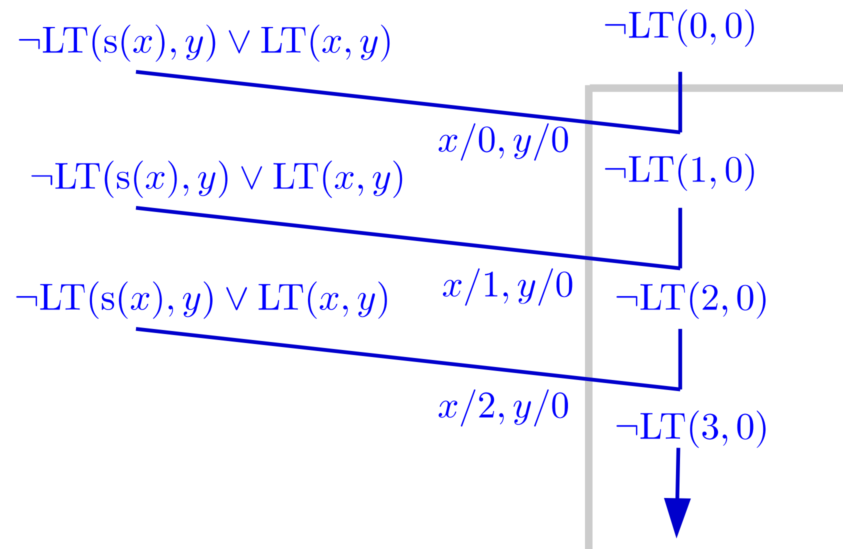
KB:

$\text{LessThan}(\text{succ}(x), y) \rightarrow \text{LessThan}(x, y)$

Query:

$\text{LessThan}(0, 0)$

Should fail since $KB \not\models \text{LessThan}(0,0)$



# Basis for Logic Programming

- Since FO Horn is undecidable it is also very expressive
- FO Horn and SLD resolution form the basis for Prolog
  - A general purpose programming language based on logic
  - Provides an intuitive language for expressing knowledge
  - Prolog is Turing-complete
  - Prolog is a form of declarative programming – you specify what the program should do not how it should do it

# Prolog

….go to Prolog slides

# Concluding Remarks

# Conclusion

- Scoped out the KRR landscape and relationship between formalisms
- Looked at propositional and first-order Horn clauses and SLD resolution
  - Empasised distinction between *Semantics* vs *Syntax*
    - *Entailment* (meaning) $S \models \alpha$
    - *Inference* (symbol manipulation) $S \vdash \alpha$
- Looked at Prolog
  - Turing complete: general purpose programming language
  - Declarative programming allows for compact representations

# Coming Weeks

- Prolog's expressivity comes with a cost
  - Efficiency issues and undecidability
  - Operational behaviour violates logical semantics; cut (!) operator, ordering of clauses.
- In coming weeks will look at more specialised logics that take a different approach to balance expressibility-computability-efficiency