Declarative / procedural

Theorem proving (like resolution) is a general domain-independent method of reasoning

Does not require the user to know how knowledge will be used

will try all logically permissible uses

Sometimes have ideas about how to use knowledge, how to search for derivations

do not want to use arbitrary or stupid order

Want to communicate to ATP procedure guidance based on properties of domain

- perhaps specific method to use
- perhaps merely method to avoid

Example: directional connectives

In general: control of reasoning

DB + rules

Can often separate (Horn) clauses into two components:

- database of facts
  - basic facts of the domain
  - usually ground atomic wffs
- collection of rules
  - extend vocabulary in terms of basic facts
  - usually universally quantified conditionals

Both retrieved by unification matching

Example:

MotherOf(jane,billy)
FatherOf(john,billy)
FatherOf(sam, john)
...
ParentOf(x,y) $\Leftarrow$ MotherOf(x,y)
ParentOf(x,y) $\Leftarrow$ FatherOf(x,y)
ChildOf(x,y) $\Leftarrow$ ParentOf(y,x)

...

Control Issue: how to use rules

Rule formulation

Consider AncestorOf in terms of ParentOf

Three logically equivalent versions:

1. $\text{AncestorOf}(x,y) \Leftarrow \text{ParentOf}(x,y)$
   $\text{AncestorOf}(x,y) \Leftarrow \text{ParentOf}(x,z) \wedge \text{AncestorOf}(z,y)$

2. $\text{AncestorOf}(x,y) \Leftarrow \text{ParentOf}(x,y)$
   $\text{AncestorOf}(x,y) \Leftarrow \text{ParentOf}(z,y) \wedge \text{AncestorOf}(x,z)$

3. $\text{AncestorOf}(x,y) \Leftarrow \text{ParentOf}(x,y)$
   $\text{AncestorOf}(x,y) \Leftarrow \text{AncestorOf}(x,z) \wedge \text{AncestorOf}(z,y)$

Back-chaining goal of AncestorOf(sam,sue) will ultimately reduce to set of ParentOf(–,–) goals

1. get $\text{ParentOf}(\text{sam},z)$: find child of Sam
   searches downward from Sam
2. get $\text{ParentOf}(z,\text{sue})$: find parent of Sue
   searches upward from Sue
3. get ParentOf(–,–): find parent relations
   searches in both directions

Search strategies are not equivalent

if more than 2 children per parent, (2) is best

Algorithm design

Example: Fibonacci numbers

1, 1, 2, 3, 5, 8, 13, 21, ...

Version 1:

$\text{Fibo}(0, 1)$

$\text{Fibo}(1, 1)$

$$\text{Fibo}(s(s(n)), x) \Leftarrow \text{Fibo}(n, y) \wedge \text{Fibo}(s(n), z) \wedge \text{Plus}(y, z, x)$$

Requires exponential number of Plus subgoals

Version 2:

$\text{Fibo}(n, x) \Leftarrow \text{F}(n, 1, 0, x)$

$\text{F}(0, c, p, c)$

$\text{F}(s(n), c, p, x) \Leftarrow \text{Plus}(p, c, s) \wedge \text{F}(n, s, c, x)$

Requires only linear number of Plus subgoals

Ordering goals

Example:

AmericanCousinOf(x,y) $\Leftarrow$
American(x) $\wedge$ CousinOf(x,y)

In back-chaining, can try to solve either subgoal first

Not much difference for

AmericanCousinOf(fred, sally)

Big difference for

AmericanCousinOf(x, sally)

1. find an American and then check to see if she is a cousin of Sally
2. find a cousin of Sally and then check to see if she is an American

So want to be able to order goals

better to generate cousins and test for American

In Prolog: order clauses, and literals in them

Notation: G :- $G_1, G_2, ..., G_n$ stands for
$G \Leftarrow G_1 \wedge G_2 \wedge ... \wedge G_n$

but goals are attempted in presented order

Commit

Need to allow for backtracking in goals

AmericanCousinOf(x,y) :-
CousinOf(x,y),
American(x)

for goal AmericanCousinOf(x,sally), may need to try American(x) for various values of x

But sometimes, given clause of the form

G :- T, S

goal T is needed only as a test for the applicability of subgoal S

In other words: if T succeeds, commit to S as the only way of achieving goal G.

so if S fails, then G is considered to have failed

- do not look for other ways of solving T
- do not look for other clauses with G as head

In Prolog: use of cut symbol

Notation: G :- $T_1, T_2, ..., T_m$, !, $G_1, G_2, ..., G_n$

attempt goals in order, but if all T_i succeed, then commit to G_i

If-then-else

Sometimes inconvenient to separate clauses in terms of unification, as in

G(zero, –) :- *method 1*
G(succ(n), –) :- *method 2*

For example, might not have distinct cases:

NumberOfParentsOf(adam, 0)
NumberOfParentsOf(eve, 0)
NumberOfParentsOf(x, 2)

want: 2 for everyone except Adam and Eve

Or cases may split based on computed property:

Expt(a, n, x) :- Even(n), (*what to do when n is even*)
Expt(a, n, x) :- Even(s(n)), (*what to do when n is odd*)

want: check for even numbers only once

Solution: use ! to do if-then-else

G :- P, !, Q.
G :- R.

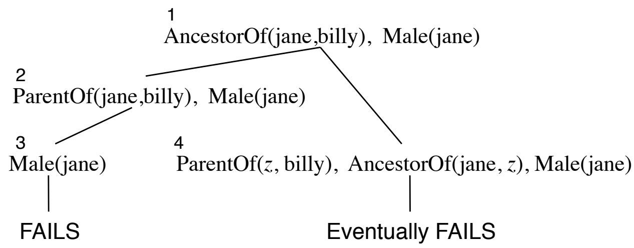
To achieve G: if P then use Q else use R

Expt(a, n, x) :- Even(n), !, (*for even n*)
Expt(a, n, x) :- (*for odd n*)

NumberOfParentsOf(adam, 0) :- !
NumberOfParentsOf(eve, 0) :- !
NumberOfParentsOf(x, 2)

Controlling backtracking

Consider a goal



So goal should be:

AncestorOf(jane,billy), !, Male(jane)

Similarly:

Member(x,l) ⇐ FirstElement(x,l)
Member(x,l) ⇐ Rest(l,l') ∧ Member(x,l')

If only to be used for testing, want

Member(x,l) :- FirstElement(x,l), !

On failure, do not try to find another x later in rest of list

Negation as failure

Procedurally: can distinguish between

- can solve goal $\neg G$
- cannot solve G

Use **not**(G) to mean goal that succeeds if G fails, and fails if G succeeds

Roughly

not(G) :- G, !, fail   /* fail if G succeeds */
not(G)   /* otherwise succeed */

Only terminates when failure is <u>finite</u>

no more resolvents vs. infinite branch

Useful when DB + rules is complete

NoParents(x) :- **not**(ParentOf(z,x))

or when method already exists for complement

Composite(n) :- **not**(PrimeNum(n))

Declaratively: same reading as $\neg$, but complications with <u>new</u> variables in G

[**not**(ParentOf(z,x)) $\supset$ NoParents(x)]   4

vs. [$\neg$ParentOf(z,x) $\supset$ NoParents(x)]   8

Dynamic DB

Sometimes useful to think of DB as a snapshot of the world that can be changed dynamically

assertions, deletions

then useful to consider three procedural interpretations for rules like

ParentOf(x,y) $\Leftarrow$ MotherOf(x,y)

1. If-needed

   Whenever have a goal matching ParentOf(x,y), can solve it by solving MotherOf(x,y)

   ordinary back-chaining, as in Prolog

2. If-added

   Whenever something matching MotherOf(x,y) is added to the DB, also add ParentOf(x,y)

   forward-chaining

3. If-removed

   Whenever something matching MotherOf(x,y) is removed from the DB, also remove ParentOf(x,y)

   keeping track of <u>dependencies</u> in DB

Interpretations (2) and (3) suggest demons

__procedures that monitor DB and fire when certain conditions are met

The Planner language

Main ideas:

1. DB of facts

   (Mother susan john)
   (Person john)

2. If-needed, if-added, if-removed procedures consisting of
   - body: program to execute
   - pattern for invocation (Mother *x* *y*)

3. Each program statement can succeed or fail
   - **(goal** *p***)**, **(assert** *p***)**, **(erase** *p***)**,
   - **(and** *s* ... *s***)**, statements with backtracking
   - **(not** *s***)**, negation as failure
   - **(for** *p* *s***)**, do *s* for every way *p* succeeds
   - **(finalize** *s***)**, like cut
   - a lot more, including all of Lisp

Example:

```
(proc if-needed (cleartable)
  (for (on x table)
     (and (erase (on x table))
          (goal (putaway x)))))

(proc if-removed (on x y)
  (print x " is no longer on " y))
```

Shift from proving conditions to making conditions hold

(if only in DB)