GSOE9210 Engineering Decisions

Victor Jauregui

vicj@cse.unsw.edu.au
www.cse.unsw.edu.au/~gs9210

Decisions under certainty and ignorance

Outline

1. Decision problem classes
2. Decisions under certainty
3. Decisions under ignorance

Decision problem classes

Decision problems can be classified based on an agent's epistemic state:

- Decisions under *certainty*: the agent knows the actual state
- Decisions under *uncertainty*:
  - Decisions under *ignorance* (full uncertainty): the agent believes multiple states/outcomes are possible; likelihoods unknown
  - Decisions under *risk*: the agent believes multiple states/outcomes are possible; likelihood information available

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Decisions under certainty

Example (Project budgeting)

You are a lead software engineer in a major software company. Your R & D team has proposed three possible projects, A, B, and C, each with a different life-time. The net profits over the life of the projects are listed in the adjacent table.

	profit ($M)
A	20
B	13
C	17

- Which project would you choose?

Complex outcomes

Project life-time cash-flows:

- A: three years, big initial set-up costs
- B: one year immediate return
- C: three years, small initial set-up costs

	cashflow ($M) Year 1	2	3
A	-10	5	25
B	13	0	0
C	-5	10	12

New perspective:

- Outcomes described by vectors: *e.g.*, for A: $(-10, 5, 25)$.
- What is more important: maximising total return, preserving cash, *etc.*?
- Which project would you choose?

Composite outcomes: Net Present Value (NPV)

- The *Net Present Value* (NPV) of a project is the value of the project at *present*
- Cash flows in the future are worth less than in the present
- Model this by a *discount rate* for each period; assume discount rate of 20%

$$\begin{aligned} NPV(\mathrm{A}) &= -10 + \frac{5}{1.2} + \frac{25}{1.2^2} &&= 11.5 \\ NPV(\mathrm{B}) & &&= 13.0 \\ NPV(\mathrm{C}) &= -5 + \frac{10}{1.2} + \frac{12}{1.2^2} &&= 11.7 \end{aligned}$$

- More generally:

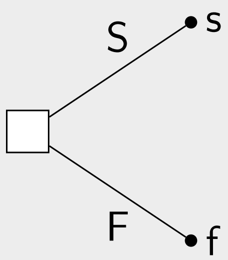
$$v(x_1, x_2, x_3) = x_1 + \frac{x_2}{1+\gamma} + \frac{x_3}{(1+\gamma)^2}$$

Decisions under certainty

Example (School fund-raising)

A school committee is looking to hold a fund-raiser. It has a choice between holding a fête or a sports day.

	s_0
S	s
F	f



- In this example:
  $\mathcal{A} = \{\mathsf{S}, \mathsf{F}\}$
  $\Omega = \{\mathsf{s}, \mathsf{f}\}$
  $\mathcal{S} = \{s_0\}$
- Which action is preferred: F or S?
- Which outcome is preferred: f or s?

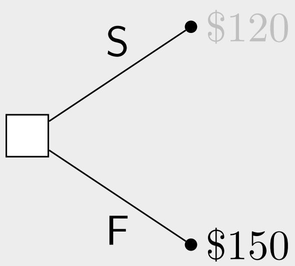
Decisions under certainty

Decisions under certainty: value functions

Example (School fund-raising)

A school committee is looking to hold a fund-raiser. It has a choice between holding a fête or a sports day. It expects to make $\$150$ for a fête but only $\$120$ for a sports day.

	s_0	V
S	$\$120$	$\$120$
F	$\$150$	$\$150$



- Value function over *outcomes*:
  $v : \Omega \to \mathbb{R}$
- In this example:
  $v(\mathsf{s}) = \$120$
  $v(\mathsf{f}) = \$150$
- Value function over *actions*; *i.e.*, $V : \mathcal{A} \to \mathbb{R}$; here $V(\mathsf{A}) = v(\omega)$, where $\omega = \omega(\mathsf{A}, s_0)$

Rational decisions

- Decision theory seeks to model *normative* (*i.e.*, *rational*) decision-making: *i.e.*, decisions ideal rational agents *ought* to make
- Which principles govern rational decision-making?

Rationality Principle 1 (Elimination)

Faced with two possible alternatives, rational agents should never choose the less preferred one.

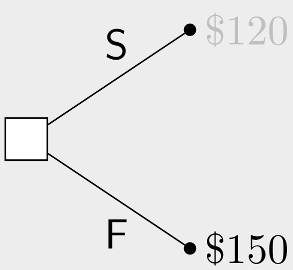
- The principle of elimination says that rational agents should eliminate less preferred actions from consideration
- A rational decision rule should satisfy this principle

Rational decisions under certainty

Corollary

Given a value function $V : \mathcal{A} \to \mathbb{R}$ over actions, rational agents should prefer action A to B iff $V(\mathsf{A}) > V(\mathsf{B})$.

	s_0	V
S	$\$120$	$\$120$
F	$\$150$	$\$150$



- Since F is preferred to S ($V(\mathsf{F}) > V(\mathsf{S})$), S is eliminated (by elimination), hence the rational choice is the remaining option: F

Corollary

A rational agent should not choose actions which are not preference maximal; *i.e.*, they should choose only actions that are preference maximal.

Outline

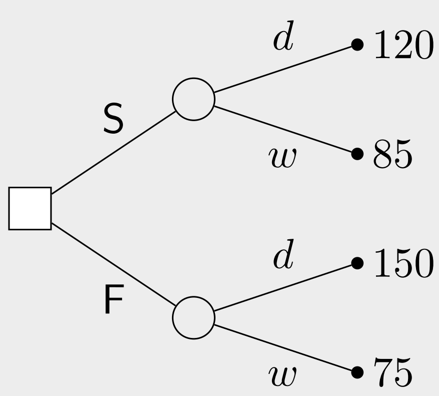
Decisions under uncertainty

Example (Uncertain school fund-raising)

Proceeds of the school fund-raiser depend on the weather; on a dry day (d) the school expects to make $\$150$ for a fête (F) but only $\$120$ for a sports day (S). However, on a wet day (w) the sports day will net $\$85$ and the fête only $\$75$.

	d	w
S	120	85
F	150	75

d day is dry
w day is wet



Decisions under uncertainty

Problem

How to assign values to uncertain actions?

	d	w
S	120	85
F	150	75

- How should $V(\mathsf{S})$ and $V(\mathsf{F})$ be defined?
- How should V (over actions) depend on v (over outcomes)?

Lotteries

Definition (Lottery)

A *lottery* over a finite set of states $\mathcal{S}$, and outcomes, or *prizes*, Ω, is a function $\ell : \mathcal{S} \to \Omega$. The lottery ℓ is written:

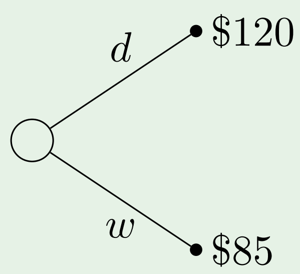
$$\ell = [s_1 : \omega_1 | s_2 : \omega_2 | \ldots | s_n : \omega_n]$$

where for each $s_i \in \mathcal{S}$, $\omega_i = \ell(s_i)$.

Example (Dry or wet?)

The uncertain situation in which the weather on a given future day is unknown represented by the lottery:

$$\ell_{\mathsf{S}} = [d : \$120 | w : \$85]$$



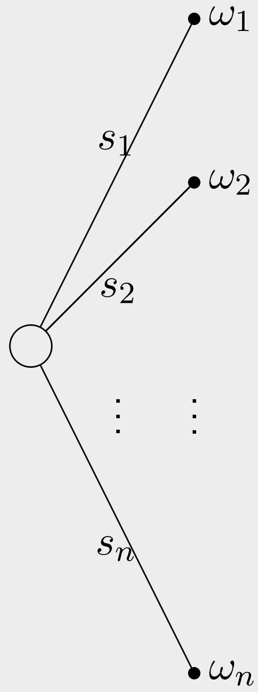
Decision problems and lotteries

$$[s_1 : \omega_1 | s_2 : \omega_2 | \ldots | s_n : \omega_n]$$

- Each uncertain action corresponds to a lottery
- Choosing an action corresponds to choosing among the lotteries on offer

Problem

The problem of evaluating actions amounts to the problem of determining how to compare and/or evaluate lotteries.



Decisions under uncertainty: ignorance

Example (Raffle)

There are four raffle tickets in a hat. Each ticket is either blue or red, but you don't know how many of each there are. Blue tickets win $\$3$; red ones lose ($\$0$). The cost of entering the raffle is $\$1$.

Exercises

- Draw the decision tree and table for this problem
- Should you draw a ticket in the raffle?
- What if you knew there were three blue tickets? Four? None?
- How many blue tickets would there have to be to make it worth entering?
- If there were n blue tickets ($0 \leqslant n \leqslant 4$), what would the prize have to be to make it worthwhile entering?

Decisions under ignorance

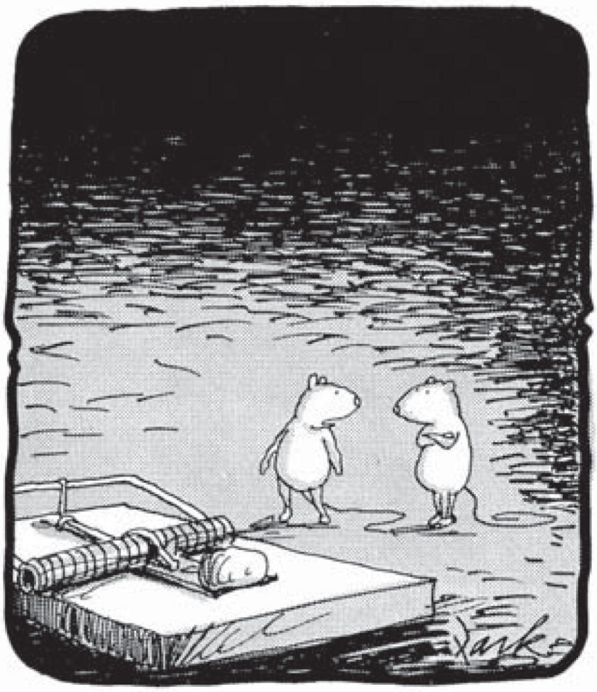
Definition (Decision rule)

A *decision rule* is a way of choosing, for each decision problem, an action or set of actions.

Rational decision rules under ignorance:

- Optimistic *MaxiMax* rule: "nothing ventured, nothing gained"
- Wald's pessimistic *Maximin* rule: "its better to be safe than sorry"
- *Hurwicz*'s mixed optimistic-pessimistic rule: use an *optimism index* α
- Savage's *miniMax Regret* rule; least *opportunity loss*
- Laplace's *principle of insufficient reason* rule

MaxiMax



"Modern technology being what it is, there's a good chance it won't work anyway."

MaxiMax associates with each action the state which yields the most preferred outcome (*i.e.*, preference maximal)

The *MaxiMax* decision rule selects the action(s) which yield a preference-maximal outcome among these

Decisions under uncertainty

Example (Uncertain school fund-raising)

Proceeds of the school fund-raiser depend on the weather; on a dry day (d) the school expects to make $\$150$ for a fête (F) but only $\$120$ for a sports day (S). However, on a wet day (w) the sports day will net $\$85$ and the fête only $\$75$.

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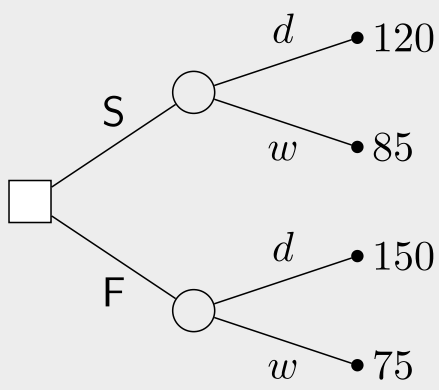

MaxiMax (MM): aim for the best

	s_1	s_2	s_3	V
A_1	6	0	4	6
A_2	2	5	1	5
A_3	4	3	2	4

- For each action find the best possible outcome of all possible cases/states; *i.e.*, for each row find the maximum value: $V_{MM}(\mathsf{A}) = M(\mathsf{A}) = \max\{v(\omega(\mathsf{A}, s)) \mid s \in \mathcal{S}\}$
- Choose the actions/rows with maximal value: A_1
- Equivalently: find the maximum value of the entire table, choose the row/action with this value
- $r_{MM}(\omega) = \arg\max\{M(\mathsf{A}) \mid \mathsf{A} \in \mathcal{A}\}$

MaxiMax

	s_1	s_2
A	10	0
B	9	9

- Which action is better?
- How could ties be broken?

	s_1	s_2	s_3	V
A_1	6	0	4	6
A_2	2	6	1	6
A_3	4	3	2	4

Risk attitudes

- *MaxiMax* is a decision rules for extreme *risk takers*
- Some agents may *prefer* risks if the favourable outcomes are sufficiently desirable
- *MaxiMax* would be a rational decision rule decision-makers with risk-taking attitudes/preferences
- In many cases it is wise to be *risk averse* (dislike risk): *i.e.*, avoid, reduce, or protect against risk
- What might a risk averse decision rule look like?

Decisions under uncertainty

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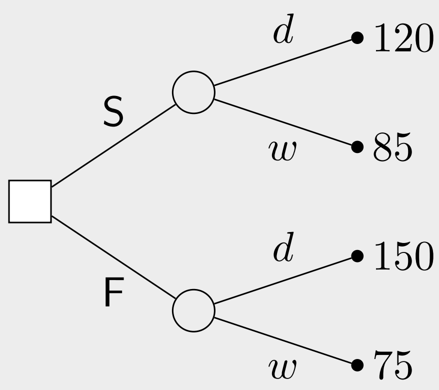

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Maximin (Mm): best in the worst case

- Assume the worst case/state will occur for each action

	s_1	s_2	s_3	V
A_1	6	0	4	0
A_2	2	5	1	1
A_3	4	3	2	2

- For each action find the worst possible outcome under all possible cases/states; *i.e.*, for each row find the minimum value: $V(\mathsf{A}) = m(\mathsf{A}) = \min\{v(\omega(\mathsf{A}, s)) \mid s \in \mathcal{S}\}$
- $m(\mathsf{A})$ is sometimes called the *security level* of action A
- Choose the action/row with the maximum of these: A_3
- $r_{Mm}(\omega) = \arg\max\{m(\mathsf{A}) \mid \mathsf{A} \in \mathcal{A}\}$

Maximin

	s_1	s_2
A	10	0
B	1	1

- Which action is better?
- How could ties be broken?

	s_1	s_2	s_3	V
A_1	6	0	4	0
A_2	2	5	2	2
A_3	4	3	2	2

Hurwicz's optimism index

	s_1	s_2	s_3	M	m	$\alpha M + (1-\alpha)m$
A_1	6	0	4	6	0	$\frac{9}{2}$
A_2	2	5	1	5	1	$\frac{8}{2}$
A_3	4	3	2	4	2	$\frac{7}{2}$

- For each action/row, find the minimum (m) and maximum (M) values
- Calculate a weighted sum based on the *optimism index* α (*e.g.*, $\alpha = \frac{3}{4}$); *i.e.*, $V(\mathsf{A}) = \alpha M(\mathsf{A}) + (1-\alpha)m(\mathsf{A}) = \frac{3}{4}M + \frac{1}{4}m$.
- Choose the row/action that maximises this value: A_1

Exercise

What happens when $\alpha = 1$? $\alpha = 0$?

MaxiMax and *Maximin*

	s_1	s_2
A	100	0
B	99	99

	s_1	s_2
A	100	4
B	5	5

	s_1	s_2
A	a_1	a_2
B	b_1	b_2

Compare problems above:

- *MaxiMax* and *Maximin* choose the same action for any values of a_1, a_2, b_1, b_2, provided $a_1 > b_1 \geqslant b_2 > a_2$ is preserved; since $M(\text{A}) = a_1 > M(\text{B})$, $m(\text{B}) > a_2 = m(\text{A})$ remain unchanged; *i.e.*, the actual numbers are irrelevant for the rules
- In this case the differences $a_1 - b_1$ and $b_2 - a_2$ are irrelevant provided $M(\text{A}) > M(\text{B})$ and $m(\text{B}) > m(\text{A})$

Decisions under uncertainty

Example (Uncertain school fund-raising)

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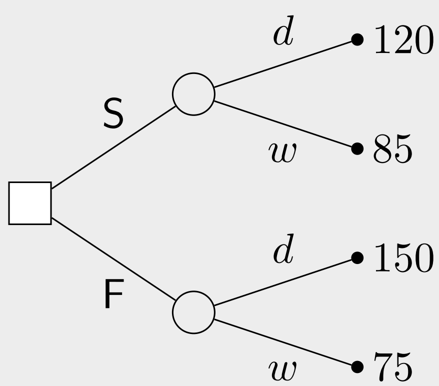

Regret

Definition (Regret)

The *regret*, or *opportunity loss*, of an outcome in a given state is the difference between the outcome's value and that of the best possible outcome for that state.

Consider the fund-raising problem discussed earlier:

	d	w
S	120	85
F	150	75
M_s	150	85

	d	w	M_R
S	30	0	30
F	0	10	10

- The maximum regret for the sports day is 30 but only 10 for the fête
- The action which minimises the maximum regret is F

miniMax Regret

	s_1	s_2	s_3
A_1	6	0	4
A_2	2	5	1
A_3	4	3	2
	6	5	4

	s_1	s_2	s_3	V
A_1	0	5	0	5
A_2	4	0	3	4
A_3	2	2	2	2

- For each column/state s, find its maximum value (M_s)
- Construct the regret table: $R(\omega) = M_s - v(\omega)$
- For each action/row find the maximum regret:
  $V(\mathsf{A}) = \max\{R(\omega(\mathsf{A}, s)) \mid s \in \mathcal{S}\}$
- Choose the row/action that minimises the regret: A_3

Laplace's insufficient reason

	s_1	s_2	s_3	V
A_1	6	0	4	$\frac{6}{3} + 0 + \frac{4}{3} = 3\frac{1}{3}$
A_2	2	5	1	$\frac{2}{3} + \frac{5}{3} + \frac{1}{3} = 2\frac{2}{3}$
A_3	4	3	2	$\frac{4}{3} + \frac{3}{3} + \frac{2}{3} = 3$

- *Assume* each state is equally likely
- For each row/action calculate the mean value: $V(A) = \frac{1}{n}v(\omega(A, s_1)) + \cdots + \frac{1}{n}v(\omega(A, s_n))$
- Choose the row/action with maximum value: A_1

Exercise

How could you simplify this decision rule?

Decision rules

	s_1	s_2	s_3	s_4	V
A_1	2	2	0	1	
A_2	1	1	1	1	
A_3	0	4	0	0	
A_4	1	3	0	0	

- For a value function V on actions, a decision rule r_V is defined by:

$$r_V(p) = \arg\max\{V(A) \mid A \in \mathcal{A}\}$$

- On the decision problem above, which rules agree (*i.e.*, choose the same actions)?

$$\begin{aligned} V_{MM}(A) &= \max\{v(\omega(A, s)) \mid s \in \mathcal{S}\} \\ V_{Mm}(A) &= \min\{v(\omega(A, s)) \mid s \in \mathcal{S}\} \\ V_{mMR}(A) &= \max\{R(A, s) \mid s \in \mathcal{S}\} \end{aligned}$$