Exercise 1. Suppose there exists a $O^*(1.2^n)$ time algorithm, which, given a graph $G$ on $n$ vertices, computes the size of a largest independent set of $G$.

Design an algorithm, which, given a graph $G$, finds a largest independent set of $G$ in time $O^*(1.2^n)$.

Exercise 2. Let $A$ be a branching algorithm, such that, on any input of size at most $n$ its search tree has height at most $n$ and for the number of leaves $L(n)$, we have

$$L(n) = 3 \cdot L(n - 2)$$

Upper bound the running time of $A$, assuming it spends only polynomial time at each node of the search tree.

Exercise 3. Same question, except that

$$L(n) \leq \max \begin{cases} 2 \cdot L(n - 3) \\ L(n - 2) + L(n - 4) \\ 2 \cdot L(n - 2) \\ L(n - 1) \end{cases}$$

Exercise 4. Consider the Max 2-CSP problem

Max 2-CSP

Input: A graph $G = (V, E)$ and a set $S$ of score functions containing

- a score function $s_e : \{0, 1\}^2 \rightarrow \mathbb{N}_0$ for each edge $e \in E$,
- a score function $s_v : \{0, 1\} \rightarrow \mathbb{N}_0$ for each vertex $v \in V$, and
- a score “function” $s_\emptyset : \{0, 1\}^\emptyset \rightarrow \mathbb{N}_0$ (which takes no arguments and is just a constant convenient for bookkeeping).

Output: The maximum score $s(\phi)$ of an assignment $\phi : V \rightarrow \{0, 1\}$:

$$s(\phi) := s_\emptyset + \sum_{v \in V} s_v(\phi(v)) + \sum_{uv \in E} s_{uv}(\phi(u), \phi(v)).$$

1. Design simplification rules for vertices of degree $\leq 2$.
2. Using the simple analysis, design and analyze an $O^*(2^{m/4})$ time algorithm, where $m = |E|$.
3. Use the measure $\mu := w_e \cdot m + (\sum_{v \in V} w_{d_G(v)})$ to improve the analysis to $O^*(2^{m/5})$. 