GSOE9210 Engineering Decisions

Victor Jauregui

vicj@cse.unsw.edu.au
www.cse.unsw.edu.au/~gs9210

Updating belief

1. Bayesian updating
   - Airline case study
2. Value of information
   - Actions which affect epistemic state
3. Sensitivity analysis

Outline

1. Bayesian updating
   - Airline case study
2. Value of information
   - Actions which affect epistemic state
3. Sensitivity analysis

Case study: capital purchase

Example (Unit purchase)

You're the chief engineer of a small commercial airline which, due to increased demand for its services, is considering buying (B) a used airliner. Another company is offering to sell one of its airliners for $\$400{,}000$. The actual value of a used airliner depends on its reliability, assessment of which would require a detailed inspection.

Question: should you purchase?

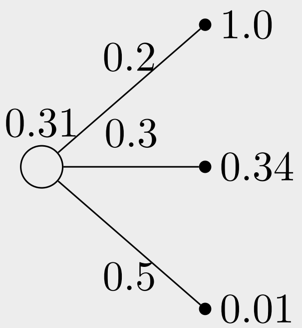
Modelling

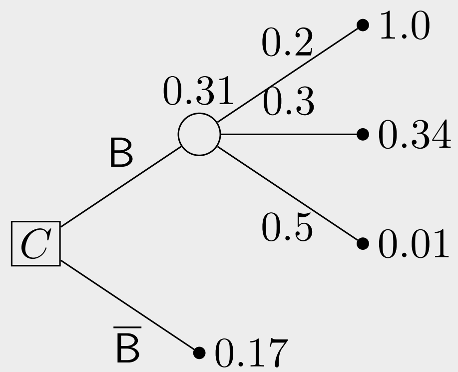
- Simplification 1: categorise used airliners as either: very reliable (vR), moderately reliable (mR), or unreliable (uR).
- Given: industry airliner reliability records

	Reliability		
	vR	mR	uR
Probability	0.2	0.3	0.5
Utility	1.0	0.34	0.01

- Simplification 2: use $\$M$ as *utiles*; actual utility should combine management's preferences about risk, financial position (*e.g.*, liquidity), customer sentiment, lost revenues, *etc.*
- Given: utility of not buying airliner—*status quo*: 0.17

Decision C (buy or not)



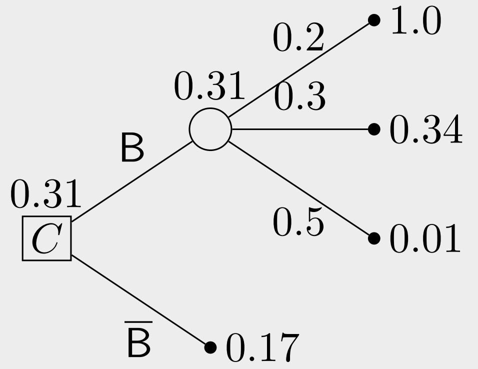


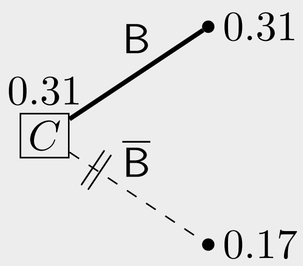
$$U(\mathsf{B}) = 0.2(1.0) + 0.3(0.34) + 0.5(0.01) = 0.31$$
$$U(\overline{\mathsf{B}}) = 0.17$$

	(0.2) vR	(0.3) mR	(0.5) uR	U
B	1.00	0.34	0.01	0.31
$\overline{\text{B}}$	0.17	0.17	0.17	0.17

Decision C

- Maximal utility principle: choose alternative with maximal expected utility
- Evaluate decision points/nodes by the maximal utility of its alternatives (*i.e.*, actions/strategies)
- The value of decision node is 0.31, because $0.31 > 0.17$; *i.e.*, $0.31 = \max\{0.17, 0.31\}$





Outline

1. Bayesian updating
   - Airline case study
2. Value of information
   - Actions which affect epistemic state
3. Sensitivity analysis

Get more information?

Example (Additional information)

You have the option to consult an aeronautical engineering firm to conduct an assessment of the airliner for $\$10{,}000$. The firm's report will be either favourable (f) or unfavourable (u).

Firm's assessment reliable?

Guess/estimate that 90% of very reliable planes receive favourable assessment; *i.e.*, $P(f|vR) = 0.9$

	... conditional on:		
Probability of:	vR	mR	uR
f	0.9	0.6	0.1
u	0.1	0.4	0.9

Bayesian revision of probabilities

- Now:

$$\begin{aligned} P(vR|f) &= \frac{P(f|vR)P(vR)}{P(f|vR)P(vR)+P(f|mR)P(mR)+P(f|uR)P(uR)} \\ &= \frac{0.9(0.2)}{0.9(0.2)+0.6(0.3)+0.1(0.5)} \\ &= \frac{0.18}{0.41} \approx 0.44 \end{aligned}$$

- Similarly:

$$P(mR|f) \approx 0.44 \qquad P(uR|f) \approx 0.12$$

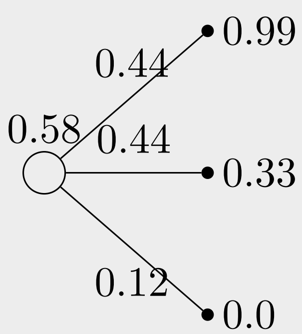
- For an unfavourable report:

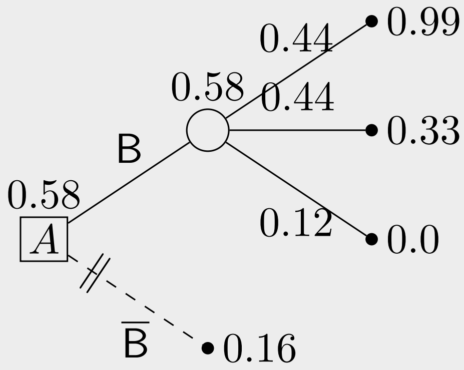
$$\begin{aligned} P(vR|u) &= \frac{0.02}{0.59} \approx 0.04 \\ P(mR|u) &\approx 0.20 \\ P(uR|u) &\approx 0.76 \end{aligned}$$

Utility adjustments

- Question: How does the report's cost ($\$10{,}000$) affect utility?
- Observation: report cost is small relative to other monetary quantities involved: the cost of the purchase is $\$400{,}000$; *i.e.*, $\$10{,}000 \ll \$400{,}000$
- Simplification 3: model effect by constant shift; *i.e.*, for report costing $\$x$ ($x \ll 400{,}000$), the change of utility is $\Delta u = \frac{x}{1{,}000{,}000}$; *i.e.*, evaluate a reliable airliner at $\$1M$
- That is, every $\$10K$ is worth 0.01 *utiles*

Decision A (report favourable)

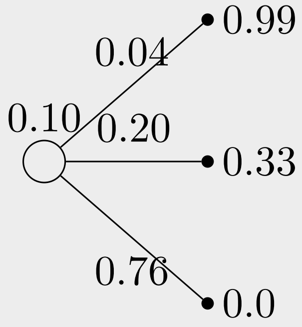


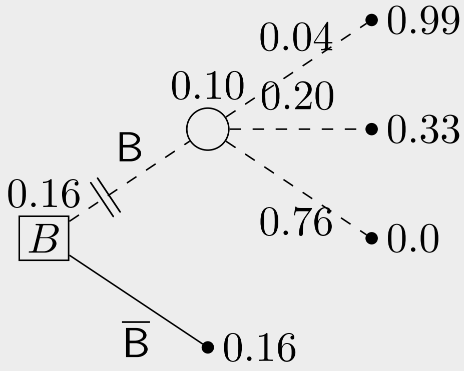


- The revised expected utility of buying the airliner is $U(\mathsf{B}) = 0.44(0.99) + 0.44(0.33) + 0.12(0.0) = 0.58$
- The utility of not buying it is $U(\overline{\mathsf{B}}) = 0.16$.

	(0.44) vR	(0.44) mR	(0.12) uR	U
B	0.99	0.33	0.0	0.58
$\overline{\mathsf{B}}$	0.16	0.16	0.16	0.16

Decision B (report unfavourable)



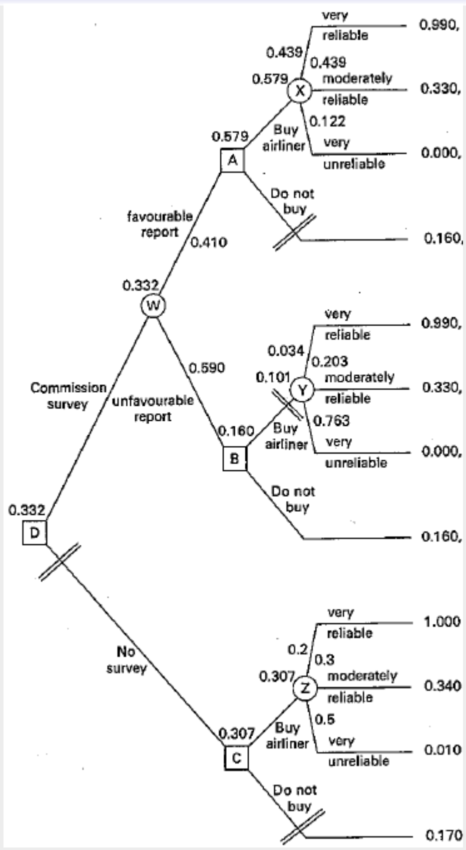


- The revised expected utility of buying the airliner is $U(\text{B}) = 0.04(0.99) + 0.20(0.33) + 0.76(0.0) = 0.10$
- The utility of not buying it is $U(\overline{\text{B}}) = 0.16$.

	(0.04) vR	(0.20) mR	(0.76) uR	U
B	0.99	0.33	0.0	0.10
$\overline{\text{B}}$	0.16	0.16	0.16	0.16

Combined decision

- Combine all three possible cases into one big decision problem
- Introduce new decision: commission survey (report), and no survey
- Introduce new event: report outcome (f or u)
- If consultant good, report likely to be good predictor of (*i.e.*, correlated to) aircraft reliability
- Consultant's increased predictive accuracy is *valuable* in making decision

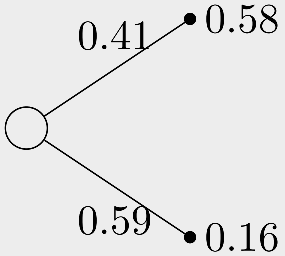


Combined decision

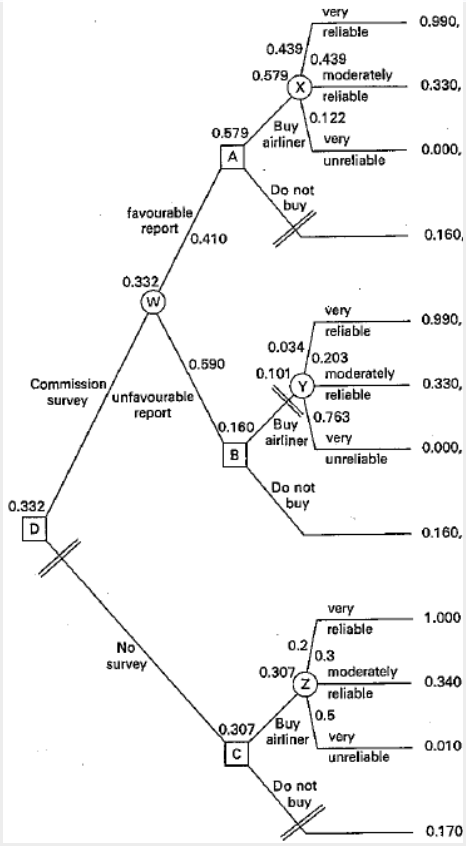
- From the denominators in the earlier calculations:

$$P(f) = 0.41$$
$$P(u) = 0.59$$

- Therefore if the report is commissioned we have the equivalent lottery:



- The U of this lottery is 0.33



Decision table

	f, vR	f, mR	f, uR	u, vR	u, mR	u, uR	U
A_1	1.0	0.34	0.01	1.0	0.34	0.01	0.31
A_2	0.17	0.17	0.17	0.17	0.17	0.17	0.17
A_3	0.99	0.33	0	0.99	0.33	0	
$\vdots$	$\vdots$	$\vdots$		$\dots$			
A_6							

where

A_1 no survey; buy airliner

A_2 no survey; don't buy airliner

A_3 commission survey; buy airliner

A_4 commission survey; don't buy

A_5 commission survey; if favourable, buy airliner; else don't buy

A_6 commission survey; if favourable, don't buy airliner; else buy

Value of information

- So the optimal policy if the report is commissioned is:

Policy C: report commissioned

If the report is favourable buy airliner, if not don't buy it.

- The value of this policy is $U(\mathsf{C}) = 0.33$, inclusive of the 0.01 fee
- The optimal policy if the report not commissioned is:

Policy $\overline{\mathsf{C}}$: report not commissioned

Buy the airliner.

- $U(\overline{\mathsf{C}}) = 0.31$
- How much is the report worth to you?
- $U(\mathsf{C}) = 0.34 - u_r \geqslant 0.31 = U(\overline{\mathsf{C}})$; *i.e.*, you should commission the report for a value/price up to $u_r = 0.03$; *i.e.*, $x \sim \$30{,}000$

Outline

Production and demand

Example (Production)

Alice is the CTO at a company and Bob is the CFO. They are discussing two possible production processes for one of its products. Measured in \$K/year, process A is expected to net $\$40$ if demand increases, $\$30$ if demand remains stable, and $\$20$ if demand falls. Process B requires a greater initial capital expenditure; it will only net $\$10$ if demand drops, and $\$40$ otherwise.
Future estimates of demand are: 20% of increasing, 30% chance of staying level, and 50% of decreasing.

Which process should Alice implement?

Example

The decision table is:

	$\frac{5}{10}$ $\downarrow$	$\frac{3}{10}$ $-$	$\frac{2}{10}$ $\uparrow$	$V_\$$
A	$\$20$	$\$30$	$\$40$	$\$27$
B	$\$10$	$\$40$	$\$40$	$\$25$

$$\begin{aligned} V_\$(\mathsf{A}) &= \tfrac{5}{10}(20) + \tfrac{3}{10}(30) + \tfrac{2}{10}(40) \\ &= 10 + 9 + 8 = \$27 \\ V_\$(\mathsf{B}) &= \tfrac{5}{10}(10) + \tfrac{3}{10}(40) + \tfrac{2}{10}(40) \\ &= 5 + 12 + 8 = \$25 \end{aligned}$$

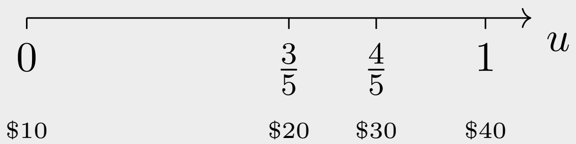
A has greater expected monetary value

Example

Alice consults Bob who advises her that, under its current financial position, the company's preferences are:

$$\$20 \sim \left[\tfrac{3}{5} : \$40 \middle| \tfrac{2}{5} : \$10\right]$$
$$\$30 \sim \left[\tfrac{4}{5} : \$40 \middle| \tfrac{1}{5} : \$10\right]$$

The company's utility for money is:



The utility table:

	$\frac{5}{10}$ $\downarrow$	$\frac{3}{10}$ $-$	$\frac{2}{10}$ $\uparrow$	U
A	$\frac{3}{5}$	$\frac{4}{5}$	1	$\frac{74}{100}$
B	0	1	1	$\frac{50}{100}$

$$\begin{aligned} U(\mathsf{A}) &= \tfrac{5}{10}(\tfrac{3}{5}) + \tfrac{3}{10}(\tfrac{4}{5}) + \tfrac{2}{10}(1) \\ &= \tfrac{1}{50}\,(15+12+10) = \tfrac{74}{100} \\ U(\mathsf{B}) &= \tfrac{5}{10}(0) + \tfrac{3}{10}(1) + \tfrac{2}{10}(1) \\ &= \tfrac{1}{50}\,(0+15+10) = \tfrac{50}{100} \end{aligned}$$

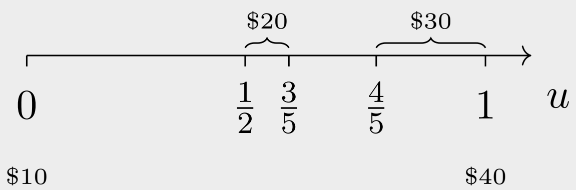
Therefore A will have greater utility.

Sensitivity analysis

Suppose Bob cannot give precise assessments on the values of \$20 and \$30, only bounds:

$$\left[\tfrac{3}{5}\$40\right] \succ \$20 \succ \left[\tfrac{1}{2}\$40\right]$$
$$\$40 \succ \$30 \succ \left[\tfrac{4}{5}\$40\right]$$

The company's utility for money is:



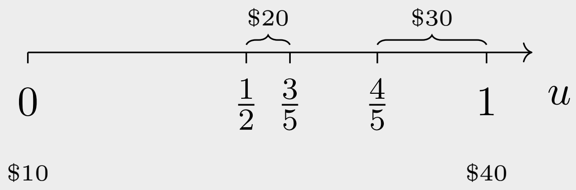
Lower bound for A:

	$\frac{5}{10}$ $\downarrow$	$\frac{3}{10}$ $-$	$\frac{2}{10}$ $\uparrow$	U
A	$\frac{1}{2}$	$\frac{4}{5}$	1	$\frac{69}{100}$
B	0	1	1	$\frac{50}{100}$

Upper bound for A:

	$\frac{5}{10}$ $\downarrow$	$\frac{3}{10}$ $-$	$\frac{2}{10}$ $\uparrow$	U
A	$\frac{3}{5}$	1	1	$\frac{80}{100}$
B	0	1	1	$\frac{50}{100}$

Sensitivity analysis



Bounds on A:

$$
\begin{aligned}
U(\mathsf{A}) &> \tfrac{5}{10}(\tfrac{1}{2}) + \tfrac{3}{10}(\tfrac{4}{5}) + \tfrac{2}{10}(1) \\
&= \tfrac{1}{100}\,(25 + 24 + 20) \\
&= \tfrac{69}{100} \\
U(\mathsf{A}) &< \tfrac{5}{10}(\tfrac{3}{5}) + \tfrac{3}{10}(1) + \tfrac{2}{10}(1) \\
&= \tfrac{1}{100}\,(30 + 30 + 20) \\
&= \tfrac{80}{100}
\end{aligned}
$$

That is:

$$\tfrac{69}{100} < U(\mathsf{A}) < \tfrac{80}{100}$$

Conclusion:
A is guaranteed to be preferred to B ($U(\mathsf{B}) = \tfrac{50}{100}$) regardless of the uncertainty over the precise preference for \$20 and \$30.

Summary

- Explored decision problems in greater depth:
  - Actions that affect epistemic state (value of information-gathering actions)
  - dealing with uncertainty in preferences (sensitivity analysis)
- Updating beliefs (epistemic state) via *Bayes*'s rule
- Value of information: cost of gathering more information versus increase in expected utility due to new information
- Sensitivity analysis:
  - decisions under imprecise preferences
  - does preference uncertainty affect a decision?