THE UNIVERSITY OF NEW SOUTH WALES

SEMESTER 2 2017

COMP6741: PARAMETERIZED AND EXACT COMPUTATION

TRIAL mid-session Quiz

1. TIME ALLOWED – 1 hour
2. READING TIME – 0 minutes
3. THIS EXAMINATION PAPER HAS 3 PAGES
4. TOTAL NUMBER OF QUESTIONS – 4
5. TOTAL MARKS AVAILABLE – 100
6. ALL QUESTIONS ARE NOT OF EQUAL VALUE. MARKS AVAILABLE FOR EACH QUESTION ARE SHOWN IN THE EXAMINATION PAPER.
7. ALL ANSWERS MUST BE WRITTEN IN INK. EXCEPT WHERE THEY ARE EXPRESSLY REQUIRED, PENCILS MAY BE USED ONLY FOR DRAWING, SKETCHING OR GRAPHICAL WORK.
8. THIS PAPER MAY BE RETAINED BY CANDIDATE.

# SPECIAL INSTRUCTIONS

9. ANSWER ALL QUESTIONS.
10. CANDIDATES MAY BRING TO THE EXAMINATION: UNSW approved calculator, all textbooks and lecture notes (handwritten or printed), private documents, etc., but no electronic material and no other electronic devices.
11. THE FOLLOWING MATERIALS WILL BE PROVIDED: answer booklet

Your answers may rely on theorems, lemmas and results stated in the lecture notes.

## 1 Search trees [10 marks]

What is the best running time upper bound you can claim for an algorithm that spends polynomial time at each node of the search tree, has no simplification rules, and has a unique 2-way branching rule that decreases a parameter $k$ **by 3 in each of the two branches**?

Possible options:

- $O^*(2^{k/3}) \subseteq O^*(1.2560^k)$;
- $O^*(\rho^k)$, where $\rho \approx 1.32472$ is the positive solution of $x^{-2} + x^{-3} = 1$;
- $O^*(3^{k/2}) \subseteq O^*(1.7321^k)$.

## 2 Self-reducibility [20 marks]

Recall that a *leaf* of a tree is a vertex with degree 1. A *spanning tree* in a graph $G = (V, E)$ is a subgraph of $G$ that is a tree and has $|V|$ vertices.

Consider the Maximum Leaf Spanning Tree problem.

| Maximum Leaf Spanning Tree | |
|---|---|
| Input: | connected graph $G$, integer $k$ |
| Parameter: | $k$ |
| Question: | Does $G$ have a spanning tree with at least $k$ leaves? |

Suppose $A$ is an algorithm solving Maximum Leaf Spanning Tree in $O^*(3.72^k)$ time.

- Design an algorithm $B$, which, for an input graph $G$, outputs a spanning tree with a largest number of leaves of $G$ in $O^*(3.72^{k^*})$ time, where $k^*$ is the largest number of leaves in any spanning tree of $G$.

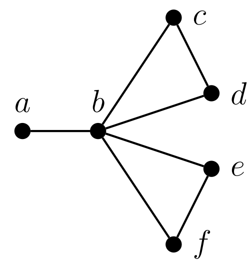
**Note**: Feel free to use polynomial-time algorithms by Prim or Kruskal for computing an arbitrary spanning tree of a graph or a spanning tree with maximum weight (when edges are weighted) without describing them.

## 3 1-Regular Vertex Deletion [35 marks]

A graph is *1-regular* if each vertex has degree 1. Consider the following problem, which asks whether it is possible to delete $k$ vertices from a graph so that it becomes 1-regular.

| 1-Regular Vertex Deletion | |
|---|---|
| Input: | A graph $G = (V, E)$ and an integer $k$ |
| Parameter: | $k$ |
| Question: | Is there a subset of vertices $S \subseteq V$ of size at most $k$, such that $G - S$ is 1-regular? |

**Example**: The following graph with $k = 2$ is a Yes-instance, since deleting $a$ and $b$ gives a graph where every vertex has degree 1.




1. Design a simplification rule that removes vertices of degree 0. [5 marks]

2. Design a simplification rule that removes vertices that have degree at least $k+2$, and show that it is sound. [10 marks]

3. Design a kernel with $O(k^2)$ vertices and edges for 1-Regular Vertex Deletion. [20 marks]

# 4 Red-Blue Dominating Set [35 marks]

Consider the Red-Blue Dominating Set problem. The input is a graph with red and blue vertices and a non-negative integer $k$, and the question is whether it is possible to select $k$ red vertices that dominate all the blue vertices.

| Red-Blue Dominating Set | |
|---|---|
| Input: | A graph $G = (V, E)$, where $V$ is partitioned into two sets $R$ (red) and $B$ (blue), and an integer $k \geq 0$ |
| Question: | Is there a subset $S \subseteq R$ with $\|S\| \leq k$ such that each vertex in $B$ has at least one neighbor in $S$? |

1. Give a Yes-instance with $k = 2$ and $|B| \geq 5$ where each vertex in $R$ has degree at most 3. [5 marks]

2. Show that Red-Blue Dominating Set is fixed-parameter tractable for the parameter $q := \Delta_R + k$, where $\Delta_R := \max_{v \in R}\{d_G(v)\}$ is the largest degree of the vertices in $R$. [30 marks]

End of Paper