COMP4418: Knowledge Representation and Reasoning—Exercise Set 1
Propositional Logic

1. Translate the following sentences into propositional logic.

   (i) If Jane and John are not in town we will play tennis

   (ii) It will either rain today or it will be dry today

   (iii) You will not pass this course unless you study

   (iv) I always drink bubble tea or soft drink after eating dinner.

   (v) If 80% of adults get fully vaccinated and COVID-19 cases begin to drop, lockdown restrictions will ease but international flights will not immediately resume.

   To do the translation you will need to

   (a) Identify a scheme of abbreviation

   (b) Identify logical connectives

2. Convert the following formulae into Conjunctive Normal Form (CNF)

   (i) $P \to Q$

   (ii) $(P \to \neg Q) \to R$

   (iii) $\neg(P \wedge \neg Q) \to (\neg R \vee \neg Q)$

   (iv) $(\neg P \to Q) \to (Q \to \neg R)$

   (v) $\neg(\neg P \vee Q) \vee (R \to \neg S)$

3. Show using the truth table method that the following inferences are valid

   (i) $P \to Q, \neg Q \models \neg P$

   (ii) $P \to Q \models \neg Q \to \neg P$

   (iii) $P \to Q, Q \to R \models P \to R$

   (iv) $P \to Q, P \to R \models P \to (Q \wedge R)$

   (v) $P \to (Q \to R) \models (P \wedge Q) \to R$

4. Repeat Question 3 using resolution. In this case we want to show:

   (i) $P \to Q, \neg Q \vdash \neg P$

   (ii) $P \to Q \vdash \neg Q \to \neg P$

   (iii) $P \to Q, Q \to R \vdash P \to R$

   (iv) $P \to Q, P \to R \vdash P \to (Q \wedge R)$

   (v) $P \to (Q \to R) \vdash (P \wedge Q) \to R$

5. Determine whether the following sentences valid (i.e., tautologies) using truth tables

(i) $((P \vee Q) \wedge \neg P) \rightarrow Q$

(ii) $((P \rightarrow Q) \wedge \neg(P \rightarrow R)) \rightarrow (P \rightarrow Q)$

(iii) $\neg(\neg P \wedge P) \wedge P$

(iv) $(P \vee Q) \rightarrow \neg(\neg P \wedge \neg Q)$

(v) $(P \vee Q) \wedge \neg(P \wedge Q)$

6. Repeat Question 5 using resolution. In this case we want to show:

   (i) $\vdash ((P \vee Q) \wedge \neg P) \rightarrow Q$

   (ii) $\vdash ((P \rightarrow Q) \wedge \neg(P \rightarrow R)) \rightarrow (P \rightarrow Q)$

   (iii) $\vdash \neg(\neg P \wedge P) \wedge P$

   (iv) $\vdash (P \vee Q) \rightarrow \neg(\neg P \wedge \neg Q)$

   (v) $\vdash (P \vee Q) \wedge \neg(P \wedge Q)$

7. Translate the following sentences to propositional logic, and use a truth table and/or resolution to determine whether the inference is valid.

   I will listen to the album "SOUR" by Olivia Rodrigo or I will watch another episode of The Queen's Gambit.
   I will not watch another episode of The Queen's Gambit.
   Therefore I will not listen to the album "SOUR" by Olivia Rodrigo.

8. Translate the following sentence to propositional logic, and use a truth table and/or resolution to determine whether it is valid (i.e. tautology).

   I will either drink too much bubble tea if I feel sick, or I will feel sick if I drink too much bubble tea.