COMP3153/9153

Algorithmic Verification

Lecture 1: Course Introduction, Logics and Automata

Acknowledgement of Country

I would like to acknowledge and pay my respect to the Bedegal people who are the Traditional Custodians of the land on which UNSW is built, and of Elders past and present.

Who are we?

I am Dr Paul Hunter. My research is on graph theory, algorithms, and formal verification.

- PhD Thesis: *Complexity and Infinite Games*
- Recent(ish) papers:
  - *Expressive completeness of MTL* (2013),
  - *When is MTL expressively complete?* (2013)

Gerald Huang will be taking tutorials.

Dr Liam O'Connor, Dr Rob van Glabbeek, and A/Prof. Peter Höfner are the former lecturers for this course.

Contacting Us

`http://www.cse.unsw.edu.au/~cs3153`

Forum

There is an ed forum available on the website. Questions about course content should typically be made there. You can ask us private questions to avoid spoiling solutions to other students.

Administrative questions should be sent to `paul.hunter@unsw.edu.au`.

Hardware Bugs: 1994 FDIV Bug

$$\frac{4195835}{3145727} =$$

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$$\frac{4195835}{3145727} = 1.33370$$

Missing entries in a hardware lookup table lead to 3-5 million defective floating point units.

Consequences:

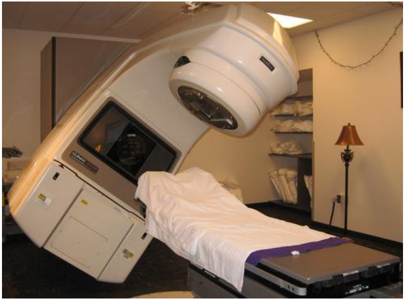
- Intel image badly damaged
- $450 million to replace FPUs.

Software Bugs: Asiana 777 Crash in 2014

Airline Blames Bad Software in San Francisco Crash

The New York Times

Software Bugs: Therac-25 (1980s)



- Radiation therapy machine.
- Two operation modes: high and low energy.
- Only supposed to use high energy mode with a shield.

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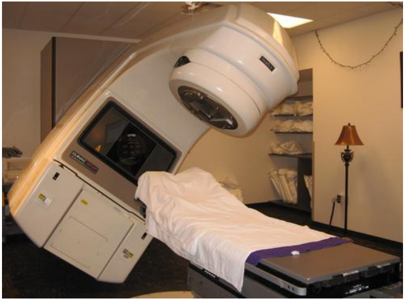

- Radiation therapy machine.
- Two operation modes: high and low energy.
- Only supposed to use high energy mode with a shield.
- Bug caused high energy mode to be used without shield.
- At least five patients died and many more exposed to high levels of radiation.

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- Sudden stalling at highway speeds.
- Bug triggered "fail-safe" mode (heh).

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Consequences:

- 75000 cars recalled.
- Cost unknown... but high.

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Consequences:

- Rocket exploded after 37 seconds.
- US$370 million cost

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- Alarm went unnoticed.
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Consequences:

- Total power failure for 7 hours, some areas up to 2 days.
- 55 million people affected
- More than US$6 billion cost

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Consequences:

- 54,000 vehicles recalled
- Cost: Have you bought a car recently?

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We'll get to more precise definitions later.

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We could try testing, but it's not exhaustive.

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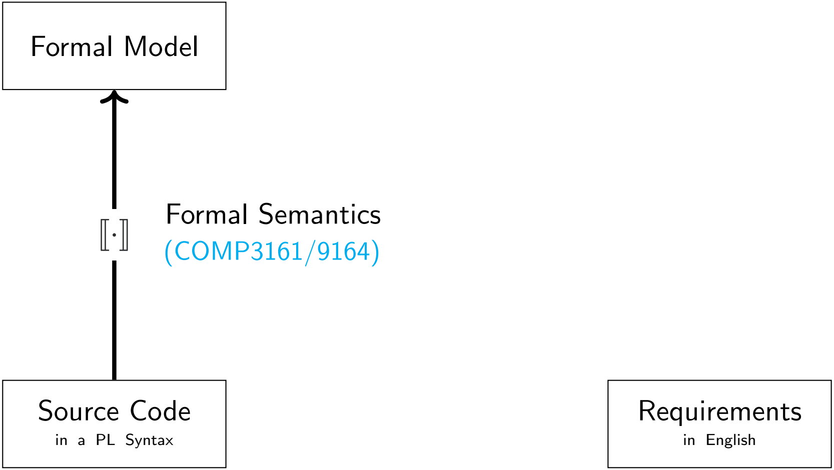
We want a rigorous and exhaustive method of verification.

Formal Verification

Source Code
in a PL Syntax

Requirements
in English

Formal Verification



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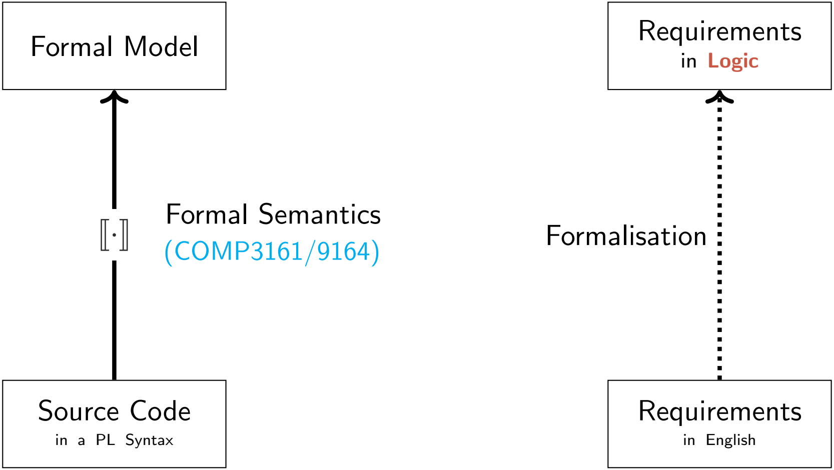

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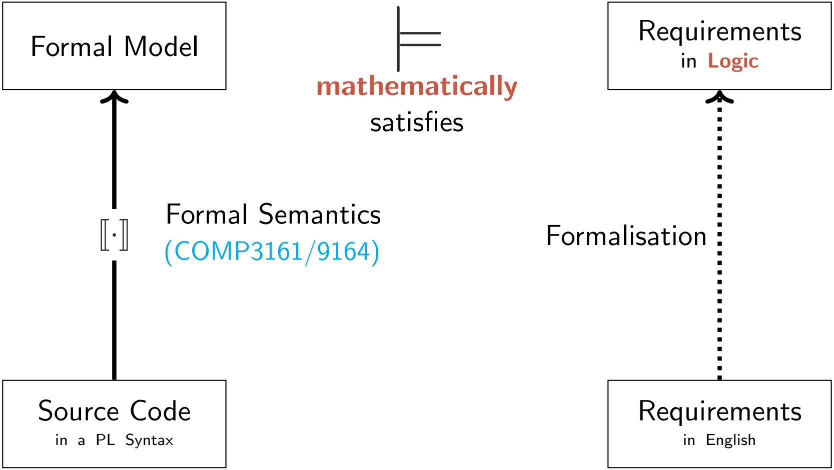

Methods of Formal Verification

Method	Automation	Speed	Expressivity	Courses
Pen/Paper Proof	None	Slow	Unbounded	COMP6721, COMP2111
Proof Assistant	Some	Medium	Unbounded	COMP4161
Model Checking	Full	Fast	Limited	This course!
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The twin foci of this course:
Model Checking and **Static Analysis**.

Model Checking

Introduced independently by Clarke, Emerson and Sistla (1980) and Queille and Sifakis (1980). **Turing Award 2007**

Formal Model

Some kind of finite automata.

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Requirements

Specify dynamic requirements with a temporal logic (Pnueli 1977 - **Turing Award 1996**).

By dynamic we mean a property of the program's executions.

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Specify dynamic requirements with a temporal logic (Pnueli 1977 - **Turing Award 1996**).

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Model checkers work by exhaustively checking the state space of the program against requirements.

Any forseeable problems with that?

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	$n = \mathbf{2}$	**3**	**4**	**5**	**6**
$m = \mathbf{2}$	6	90	2520	113400	$2^{22.8}$
3	20	1680	$2^{18.4}$	$2^{27.3}$	$2^{36.9}$
4	70	34650	$2^{25.9}$	$2^{38.1}$	$2^{51.5}$
5	252	$2^{19.5}$	$2^{33.4}$	$2^{49.1}$	$2^{66.2}$
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$$\frac{(nm)!}{m!^n}$$

State Space Explosion

There are many techniques to make model checking a more tractable problem, such as symbolic and bounded model checking, SAT-based techniques, and abstraction/refinement. We will examine these techniques throughout the course.

Tools

- SPIN, an explicit LTL model checker used for protocols, which uses heuristics to control state space.
- nuSMV, a symbolic model checker using binary decision diagrams.
- SLAM and CBMC, which are SAT-based tools using bounded model checking.

Static Analysis

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Example (Static Invariants)

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Based on the abstract interpretation technique of Cousot and Cousot (1977). We'll look at this around Week 6, but:

Key Idea

Abstract from *specific values* to *classes of values*, increasing the non-determinism of the program but making it easier to analyse possible effects of the program.

Tools: ASTREE, Absint, Coverity, Grammatech, Polyspace, PVS-Studio, Goanna etc. etc.

Course schedule

A (very) tentative course schedule, subject to change:

Week 1	Background, logic, automata	Assmt 1 released
Week 2	Model checking, Safety and Liveness	
Week 3	Tool: Spin	Assmt 1 due
Week 4	Simulation & Bisimulation	Assmt 2 released
Week 5	Static analysis, Tool: Skink	Assmt 2 due
Week 6	Flexibility week	Assmt 3 released
Week 7	Symbolic Model Checking	
Week 8	Binary Decision Diagrams	Assmt 3 due
Week 9	Timed automata and languages	Assmt 4 released
Week 10	Tool: Uppaal	Assmt 4 due

What do we expect?

Maths

This course uses a significant amount of *discrete mathematics*. You will need to be reasonably comfortable with *logic*, *set theory* and *induction*. MATH1081 ought to be sufficient for aptitude in these skills, but experience has shown this is not always true.

What do we expect?

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Programming

We expect you to be familiar with imperative programming languages like C. Course assignments may require some programming in modelling languages. Some self-study may be needed for these tools.

Assessment

There are **four** homework assignments for this course.

The final assessment is made up of your assignments plus the final exam, with equal weighting between all assignments and the exam.

Resources

Lecture Recordings

In previous years, no recordings were made available for this course. I will endeavour make them available this year, however their quality and availability is not guaranteed.

Lectures are intended to be an interactive experience – I will be delivering them in real-time.

The only way to ensure you have the best lecture experience for this course is to attend the lectures!

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Textbooks

This course follows more than one textbook. Each week's slides will include a bibliography. A list of books is given in the course outline, all of the books listed are available from the library.

Logic

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Definition

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Example (Propositional Logic Syntax)

- A set of atomic propositions $\mathcal{P} = \{a, b, c, \ldots\}$
- An inductively defined set of formulae:
  - Each $p \in \mathcal{P}$ is a formula.
  - If P and Q are formulae, then $P \wedge Q$ is a formula.
  - If P is a formula, then $\neg P$ is a formula.

(Other connectives are just sugar for these, so we omit them)

Semantics

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Semantics are a mathematical representation of the meaning of a piece of syntax. There are many ways of giving a logic semantics, but we will use models.

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Example (Propositional Logic Semantics)

A model for propositional logic is a valuation $\mathcal{V} \subseteq \mathcal{P}$, a set of "true" atomic propositions. We can extend a valuation over an entire formula, giving us a satisfaction relation:

$$\begin{array}{lcl} \mathcal{V} \models p & \Leftrightarrow & p \in \mathcal{V} \\ \mathcal{V} \models \varphi \land \psi & \Leftrightarrow & \mathcal{V} \models \varphi \text{ and } \mathcal{V} \models \psi \\ \mathcal{V} \models \neg\varphi & \Leftrightarrow & \mathcal{V} \not\models \varphi \end{array}$$

We read $\mathcal{V} \models \varphi$ as $\mathcal{V}$ "satisfies" φ.

Automata

We will model our computations using finite automata.

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Definition

A finite automata (FA) is a quintuple $(Q, q_0, \Sigma, \delta, F)$ where:

- Q is a finite set of states.
- $q_0 \in Q$ is the initial state.
- Σ is a finite set of actions called an alphabet.
- δ is a transition relation $Q \times \Sigma \to 2^Q$.
- $F \subseteq Q$ is a set of final states.

A FA is called deterministic iff δ is a function, i.e.

$$\forall (s, a) \in Q \times \Sigma.\ |\delta(s, a)| \leq 1$$

Example: binary strings ending with double zero

Automata

A run from an automata A is a sequence of transitions:

$$q_0 \xrightarrow{a_1} q_1 \xrightarrow{a_2} \cdots \xrightarrow{a_{n-1}} q_{n-1} \xrightarrow{a_n} q_n$$

This run can also be written $q_0 \xrightarrow{a_1 a_2 \ldots a_n} q_n$ or, if we don't care about the actions $q_0 \xrightarrow{\star} q_n$.

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The language $\mathcal{L}(A)$ of an automata A is all sequences of actions (words) whose runs end in the set of final states F:

$$\mathcal{L}(A) = \{w \in \Sigma^* \mid q_0 \xrightarrow{w} q, q \in F\}$$

Non-determinism

Non-deterministic finite automata can be converted to deterministic finite automata, by using sets of NFA states as the set of states for the DFA (the subset construction).

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ε-transitions

We can enrich NFAs with transitions that do not have actions (or equivalently, transitions with the empty word ε as their action) without affecting expressiveness. Subset construction still works.

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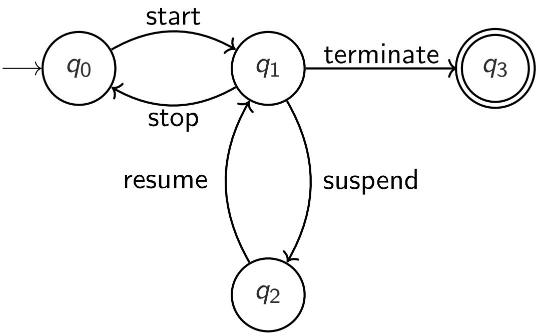
Thus,

$$\mathsf{DFA} = \mathsf{NFA} = \mathsf{NFA}^{\varepsilon}$$

Modelling with Automata



What sort of runs can this automata produce?

Intersection of Languages

Problem

Let A be a FA such that $\mathcal{L}(A)$ is the set of strings with an even number of as.

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Let B be a FA such that $\mathcal{L}(B)$ is the set of strings with an odd number of bs.
How can we combine A and B into a new automata C such that $\mathcal{L}(C) = \mathcal{L}(A) \cap \mathcal{L}(B)$?

(try to come up with a general technique for any automata)

Intersection of Languages

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Let A be a FA such that $\mathcal{L}(A)$ is the set of strings with an even number of $\mathtt{a}$s.
Let B be a FA such that $\mathcal{L}(B)$ is the set of strings with an odd number of $\mathtt{b}$s.
How can we combine A and B into a new automata C such that $\mathcal{L}(C) = \mathcal{L}(A) \cap \mathcal{L}(B)$?
(try to come up with a general technique for any automata)

We need to create the product of two automata.

Automata Product

Definition

The product of two automata
$A_1 = (Q_1, q_0^1, \Sigma_1, \delta_1, F_1)$ and
$A_2 = (Q_2, q_0^2, \Sigma_2, \delta_2, F_2)$
is defined as: $(Q, q_0, \Sigma, \delta, F)$ where:

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- $Q = Q_1 \times Q_2$
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- $\delta(\ (q_1, q_2)\ , a) =$

$$\begin{cases} \{(q_1', q_2') \mid q_1' \in \delta_1(q_1, a), q_2' \in \delta_2(q_2, a)\} & \text{if } a \in \Sigma_1 \cap \Sigma_2 \\ \{(q_1', q_2) \mid q_1' \in \delta_1(q_1, a)\} & \text{if } a \in \Sigma_1 \setminus \Sigma_2 \\ \{(q_1, q_2') \mid q_2' \in \delta_2(q_2, a)\} & \text{if } a \in \Sigma_2 \setminus \Sigma_1 \end{cases}$$

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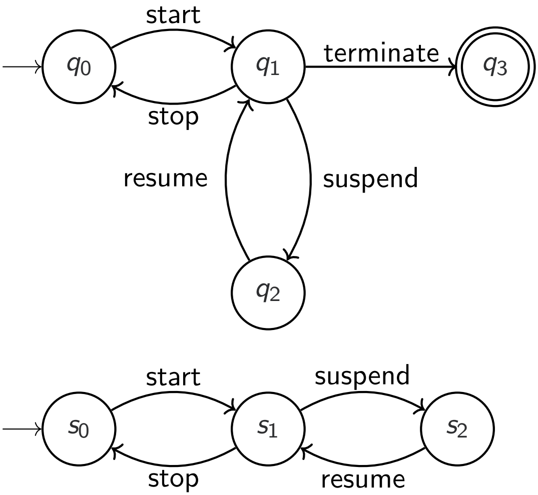
- $F = F_1 \times F_2$

Task and Scheduler



Products can encode communication. Compute the product of these two processes.

Integer Variables

Problem

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Does this affect the expressivity of automata?

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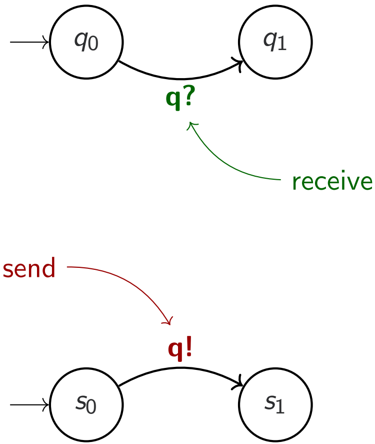
No. We can encode the integers as automata and use synchronisation.

Message passing



Different tools offer broadcast or unicast communication. **Check the manual!**

Bibliography

Propositional Logic:

- Huth/Ryan: Logic in Computer Science, Section 1
- Bayer/Katoen: Principles of Model Checking, Appendix A3

Automata:

- Sipser: Introduction to the Theory of Computation, sections 1.1 and 1.2
- Kozen: Automata and Computability, Sections 3-5